A Modified Global Convergence of Generalized Augmented Lagrange Method in Nonlinear Programming

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Abstract:

In this paper we investigated new algorithm of Augmented Lagrang-method to solve constrained optimization. The new proposed method satisfied global convergence and it is too effective when compared with other established algorithm to solve standard constrained problem.

تطوير التقارب الشامل لطريقة لاكرانج– المضخمة المعممة في البرمجة اللاخطية

الملخص

في هذا البحث تم استحداث دالة ربط جديدة لدالة لاكرانج– المضخمة الجديدة وتم برهنتها بانها تحقق شرط التقارب الشامل وانها ذات فعالية عالية عند مقارنتها مع مثيلاتها من الطرق السابقه.

1. Introduction :

The general constrained minimization problem
$$minimize\ f(x)\dots(1)$$
Subject to
$$g_i(x) \leq 0 \qquad\qquad i = 1,\dots\dots,n\dots(2)$$
$$h_i(x) = 0 \qquad\qquad i = 1,\dots\dots,k\dots(3)$$
where x is an n-dimensional vector and the functions f(x) , $g_i(x)$, i=1,… n and $h_i(x)$, i=1 ,… k are continuous *an*d usually as-summed to possess continuous second partial derivatives.(Al Bayati , 2013)

There exits an important class of methods to solve the general constrained optimization. This class of methods seeks the solution by replacing the original constrained problem with a sequence of unconstrained sub problems in which the objective function is formed by the original objective of the constrained optimization plus additional 'penalty' terms. The 'penalty' terms are made up of constraint functions

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multiplied by a positive coefficient. By making this coefficient larger and larger a length optimization of the sequential unconstrained sub problems, we force the minimize of the objective function closer and closer to the feasible region of the original constrained problem. However, as the penalty coefficient grows to be too large, the objective function of the unconstrained optimization sub problem may become ill conditioned, thus, making the optimization of the sub problem difficult. This issue is avoided, after the proof of convergence, by the so called 'Augmented Lagrange method' in which an explicit estimate of the Lagrange multipliers λ, μ is included in the objective. (Buys, 1972)

2.The Lagrange method :

Lagrange multipliers play a crucial role in the study of constrained optimization. On the one hand, the conditions imposed on the Lagrange multipliers are always an integral part of various necessary and sufficient conditions and, on the other, they provide a natural connection between constrained and corresponding unconstrained optimization problems; each individual Lagrange multiplier can be interpreted as the rate of change in the objective function with respect to changes in the associated constraint function (Flecher , 1987).

3.Augmented Lagrange Multiplier Method :
3.1 Mixed Equality–Inequality-Constrained Problems

Consider the following general optimization problem:

$$\text{minimize} \quad f(x) \quad \quad \ldots (4)$$

subject to

$$g_j(x) \leq 0 \qquad \qquad , j = 1,2,\ldots\ldots\ldots m \quad \ldots (5)$$

$$h_j(x) = 0 \qquad \qquad , j = 1,2,\ldots\ldots\ldots \rho \ldots (6)$$

This problem can be solved by combining the procedures of the two preceding sections.

The augmented Lagrangian function, in this case, is defined as

$$A(x,\lambda,r_k) = f(x) + \sum_{j=1}^{m} \lambda_j \alpha_j + \sum_{j=1}^{\rho} \lambda_{m+j} h_j(x) + r_k \sum_{j=1}^{m} \alpha_j^2 + r_k \sum_{j=1}^{\rho} h_j^2(x) \ldots (7)$$

where αj is given by

$$\alpha_j = \max\left\{ g_j(x), -\frac{\lambda_j}{2r_k} \right\}$$

The solution of the problem stated in Eqs. (4) to (6) can be found by minimizing the function A, defined by Eq. (7), as in the case of equality-constrained problems using the update formula

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} + 2r_k \max\left\{g_j(x), -\frac{\lambda_j^{(k)}}{2r_k}\right\}, \qquad j = 1,2,\ldots\ldots m \qquad \ldots (8)$$

$$\lambda_{m+j}^{(k+1)} = \lambda_{m+j}^{(k)} + 2r_k h_j(x), \qquad j = 1,2,\ldots\ldots\rho \ldots (9)$$

The ALM method has several advantages. As stated earlier, the value of r_k need not be increased to infinity for convergence. The starting design vector, x(1), need not be feasible. Finally, it is possible to achieve $g_j(x)=0$ and $h_j(x) = 0$ precisely and the nonzero values of the Lagrange multipliers ($\lambda_j \neq 0$) identify the active constraints automatically. (Rao,2009)

3.2 Outlines of The Standard Augmented Lagrange Algorithm
1. choose a tolerance $e = 10^{-5}$,starting point $x_0=0$, initial penalty parameter $r_0=1$, and initial Lagrange multipliers $\lambda_0=0$
2. Perform unconstrained optimization on the augmented Lagrangian function

3. set $\lambda_j^{(k+1)} = \lambda_j^{(k)} + 2r_k \max\left\{g_j(x), -\frac{\lambda_j^{(k)}}{2r_k}\right\}, \qquad j = 1,2,\ldots\ldots m$

And $\lambda_{m+j}^{(k+1)} = \lambda_{m+j}^{(k)} + 2r_k h_j(x), \qquad j = 1,2,\ldots\ldots\rho$

4. Increase $r_{k+1} = 2r_k \qquad if \qquad \|\lambda_{k+1} - \lambda_k\| \prec 0.5$

5. Check the convergence criteria. If $\|x^*_{k+1} - x^*_k\| \prec \varepsilon$, then stop. Otherwise, set $x_0 = x^*_k$ and return to Step 2.
(Rao,2009)

4.Augmented Lagrange Multiplier Method(PHRAUG):
The constraints defined by h(x) = 0 and g(x) ≤ 0 will be included in the augmented Lagrangian definition .Given set $\Omega = \left\{x \in IR^n | \underline{h}(x) = 0, \underline{g}(x) \leq 0\right\}$. μ Є IR^p , μ ≥ 0 ,we define the Powell-Hestenes–Rockafellar(PHR),(Buys,1972),(Conn,2000),(Hestenes,,1969),(polyak,1992)and(Rockafeller,1973)augmented Lagrangian defiend by:

$$L_\rho(x,\lambda,\mu) = f(x) + \frac{\rho}{2}\left[\sum_{i=1}^{m}\left(h_i(x) + \frac{\lambda_i}{\rho}\right)^2 + \sum_{i=1}^{\rho}\max\left\{0, g_i(x) + \frac{\mu_i}{\rho}\right\}^2\right] \quad \ldots (10)$$

PHP-like augmented lagrangian methods are based on the iterative minimization of $L_\rho(x,\lambda,\mu)$ with respect to x Є Ω followed by convenient updates of λ, μ and ρ. (Birgin ,2010) .

4.1.Algorithm :
Step 1: choose a tolerance $e = 10^{-5}$,starting point $x_0=0$, initial penalty parameter $r_0=1$, and initial Lagrange multipliers $\lambda_0=0$

step 2 :   Perform unconstrained optimization on the augmented Lagrangian function

Step 3: set

$$\lambda^{k+1} = \overline{\lambda}^k + \rho_k h(x^k)$$

and

$$\mu^{k+1} = (\overline{\mu}^k + \rho_k g(x^k))_+$$

Step 4: Increase $r_{k+1} = 2r_k$ *if* $\left\| \lambda_{k+1} - \lambda_k \right\| \prec 0.5$

Step 5 :   Check the convergence criteria. If $\left\| x^*_{k+1} - x^*_k \right\| \prec \varepsilon$, then stop. Otherwise, set $x_0 = x^*_k$ and return to Step 2.

5. New Modified Barrier Augmented Lagrangian Method(MBAUG) :

we develop a new method for solving constrained nonlinear optimization problems involving both inequality and equality constraints. Our method is a combination of the augmented Lagrangian method for equality constraints of [Hestenes (Hestenes,1969) and Powell (Powell ,1969)]with a modified barrier function (MBF) method of Polyak(Polyak ,1992). Variants of the latter method have been considered by Breitfeld and Shanno(Breifeld,1994) and Conn et al.(Conn , 1997). Since the modified barrier function can be viewed as an interior augmented Lagrangian, the MBAUG method can be viewed as an interior-exterior augmented Lagrangian method .The idea of combining a barrier function and a penalty function approach to solve constrained optimization problems with both inequality and equality constraints was suggested nearly thirty years ago by Fiacco and McCormick.(Yuzefovich , 1999)

The new modified Barrier Augmented Lagrangian multiplier Method (MBAUG)

$$BAL_\rho(x,\lambda,\mu) = f(x) + \frac{\rho^n}{n}\left[\sum_{i=1}^{m}\left(h_i(x) + \frac{\lambda_i}{2\rho^n} \right)^2 - \sum_{i=1}^{l} 2\log\left(\left| g_i(x) + \frac{\mu_i}{2\rho^n} \right| \right) \right]\dots(11)$$

Barrier augmented lagrangian methods are based on the iterative minimization of $BAL_\rho(x,\lambda,\mu)$ with respect to $x \in \Omega$ such that $\Omega = \left\{ x \in IR^n | \underline{h}(x) = \mathbf{0}, \underline{g}(x) \leq \mathbf{0} \right\}$ followed by convenient updates of λ, μ and ρ.

5. 1. Algorithm :
Step 1 : Start point x_0 initial of the feasible , n is scalar , $H_1 = I$, initial parameter $\rho_0 = 1$, initial Lagrange multiplier λ_0 , μ_0 and $\mathcal{C} = \mathbf{1 \times 10^{-5}}$.

Step 2 : Set k=1 $d_1 = -H_1 g_1$

Step 3 : Perform unconstrained optimization on the augmented lagrangian function of eq. (10) .

Set $d_k = -H_k g_k$ where H_k is BFGS method.

Step 4 : Set $x_{k+1} = x_k + \lambda_k d_k$ where λ is satisfied wolfe condition .

Step 5 :Check the convergence criteria $\left\| x^*_{k+1} - x^*_k \right\| \prec \varepsilon$ stop

Otherwise

$$\lambda^{k+1} = \frac{1}{n}\left(\overline{\lambda}^k + 2\rho_k^n \, h(x^k)\right)$$

and

$$\mu^{k+1} = \frac{1}{n}\left(\frac{4\rho^{2n}}{\overline{\mu}^k + 2\rho_k^n \, g(x^k)}\right)$$

Step 6 : set $x_0 = x_k$ and k=k+1 return to Step 3.

6. The Convergence Analysis of The New Modified Barrier Augmented Lagrangian Multiplier Method (MBAUG) :

The convergence analysis of augmented Lagrangian method is similar to that of the quadratic penalty method, but significantly more complicated because there are three parameters λ , μ ,p instead of just one. As a straightforward generalization of the previous method, we can define:

$$f(x,\lambda_+,\mu_+,\lambda,\mu,\rho) = \begin{bmatrix} \nabla f(x) + \lambda_+ \nabla h(x) - \mu_+ \nabla g(x) \\ \left(\lambda_+ + \frac{\lambda}{n}\right) + \frac{2\rho^n}{n} h(x) \\ n\mu_+ m + 2n\mu_+ \rho^n g(x) - 4\rho^{2n} \end{bmatrix} \quad \text{...(12)}$$

and solve for (x,λ_+) , (x,μ_+) regarding λ ,μ and ρ as parameters . first of all assuming as usual that x* , λ* , μ* ,lagrange multiplier pair ,

$$f(x^*,\lambda^*,\mu^*,\lambda,\mu,\rho) = \begin{bmatrix} \nabla f(x^*) + \lambda^* \nabla h(x^*) - \mu^* \nabla g(x^*) \\ \left(\lambda^* + \frac{\lambda}{n}\right) + \frac{2\rho^n}{n} h(x^*) \\ n\mu^* m + 2n\mu^* \rho^n g(x^*) - 4\rho^{2n} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{...(13)}$$

For all ρ>0 . moreover , the Jacobean of f (with respect to the variables x , λ_+ , μ_+) is

$$J(x,\lambda_+,\mu_+,\lambda,\mu,\rho) = \begin{bmatrix} \nabla^2 l(x,\lambda_+,\mu_+) & \nabla h(x) & -\nabla g(x) \\ \frac{2\rho^n}{n} \nabla h(x) & I & 0 \\ \rho^n \nabla g(x) & 0 & nmI + 2nI\rho^n g(x) \end{bmatrix} \quad \text{...(14)}$$

Assuming x* is a nonsingular point of the NLP , and using the sufficient condition the matrix

$$J(x^*,\lambda^*,\mu^* ,\lambda,\mu,\rho) = \begin{bmatrix} \nabla^2 l(x^*,\lambda^*,\mu^*) & \nabla h(x^*) & -\nabla g(x^*) \\ \dfrac{2°\rho^n}{n}\nabla h(x^*) & I & 0 \\ \rho^n \nabla g(x^*) & 0 & 2nI\rho^n g(x^*) \end{bmatrix} \dots(15)$$

As $\mu \to 0$, therefore there exists $\hat\mu > 0$ such that $J(x^*,\lambda^*,\mu^* ,\lambda,\mu,\rho)$ is nonsingular for all $\mu \in [0,\hat\mu]$. The implicit function theorem then implies that there exists a neighborhood N of $\mu^* \lambda^*$ such that there exist function x, λ_+ and x, μ_+ ,defined on $N \times [0,\hat\mu]$ such that

-$x(\lambda^*,\rho) = x^*$, $\lambda_+(\lambda^*,\rho) = \lambda^*$ $for\ all\ \rho \in [0,\hat\rho]$

- $x(\mu^*,\rho) = x^*$, $\mu_+(\mu^*,\rho) = \mu^*$ $for\ all\ \rho \in [0,\hat\rho]$

- for all λ , $\mu \in N$, $\rho \in [0,\hat\rho]$,

$f(x(\lambda,\mu,\rho),\lambda_+(\lambda,\rho),\mu_+(\mu,\rho),\lambda,\mu,\rho) = 0$

Then the function x , λ_+ , μ_+ satisfy

$$\nabla f(x(\lambda,\mu,\rho)) + \lambda_+(\lambda,\rho)\nabla h(x(\lambda,\rho)) - \mu_+(\mu,\rho)\nabla g(x(\mu,\rho)) = 0 \quad \dots (16)$$

$$\left(\lambda_+(\lambda,\rho) + \frac{\lambda}{n}\right) + \frac{2\rho^n}{n}h(x(\lambda,\rho)) = 0 \qquad \dots (17)$$

$$n\mu_+(\mu,\rho)m + 2n\mu_+(\mu,\rho)\rho^n g(x(\mu,\rho)) - 4\rho^{2n} = 0 \qquad \dots (18)$$

solving (17) , (18) , $\lambda_+(\lambda,\rho)$ and $\mu_+(\mu,\rho)$ yield

$$\lambda_+(\lambda,\rho) = \frac{-1}{n}\left(\lambda + 2\rho^n h(x)\right)$$

and

$$\mu_+(\mu,\rho) = \frac{1}{n}\left(\frac{4\rho^{2n}}{\mu + 2\rho^n g(x)}\right)$$

substituting this in to (16)
$\nabla f(x(\lambda,\mu,\rho)) - 1/n\ \lambda + 2\rho^{\wedge}n\ h(x(\lambda,\rho))\nabla h(x(\lambda,\rho)) - 1/n\ ((4\rho^{\wedge}2n)/(\mu + 2\rho^{\wedge}n\ g(x(\mu,\rho))))\nabla g(x(\mu,\rho)) = 0 \dots(19)$
rearranging the last equation shows that
$\nabla L(x(\lambda,\mu,\rho)) = 0 \qquad \dots (20)$
in other words , $x(\lambda,\rho),x(\mu,\rho)$ a stationary point of $L(x(\lambda,\mu,\rho))$ for each $\lambda \in N$, $\mu \in N$ and each $\rho \in [0,\hat\rho]$,

since

$$\nabla^2 L\big(x(\lambda,\mu,\rho),\lambda,\mu,\rho\big) = \nabla^2 l\big(x(\lambda,\mu,\rho),\lambda_+(\lambda,\rho),\mu_+(\mu,\rho)\big)$$

$$-\frac{2}{n}\rho^n\nabla h(x(\lambda,\rho))\nabla h(x(\lambda,\rho))^T - \frac{8\rho^{3n}\nabla g(x(\mu,\rho))\nabla g(x(\mu,\rho))^T}{(\mu + 2\rho^n g(x(\mu,\rho)))^2}$$

...(21)

and $x(\lambda,\rho)\rightarrow x^*$, $\lambda(\lambda,\rho)\rightarrow\lambda^*$, $x(\mu,\rho)\rightarrow x^*$, $\mu(\mu,\rho)\rightarrow\mu^*$ as $\lambda\rightarrow\lambda^*$, $\mu\rightarrow\mu^*$. it is straightforward to show that

$$\nabla^2 L\big(x(\lambda,\mu,\rho),\lambda,\mu,\rho\big) = 0$$

Is positive definite for λ, μ sufficiently close to λ^*, μ^* and for ρ sufficiently small we have therefore proved the following theorem .

6.1. Theorem:

Suppose *f: $R^n\rightarrow R$ and c:$R^n\rightarrow R^m$ are twice continuously differentiable and x^* is a local minimizes of the NLP*

$$minimize\ f(x)$$

Subject to

$$g_i(x) \leq 0 \qquad\qquad i = 1,\ldots\ldots,n\ldots(22)$$
$$h_i(x) = 0 \qquad\qquad i = 1,\ldots\ldots,k$$

If x* is a nonsingular point and λ^* is the corresponding lagrange multiplier , then there exists $\hat{\rho} > 0$, $\epsilon > 0$ and a function

$x: N \times [0,\hat{\rho}] \rightarrow R^n$, *N=Bε(λ*)*with the following properties:

1- *x* is continuously differentiable.
2- x(λ^*,ρ)=x* and x(μ^*,ρ)= x* for all $\rho \in [0,\hat{\rho}]$
3- x(λ^*,ρ)=x* and x(μ^*,ρ) is the unique local minimize of$\nabla L(x(\lambda,\mu,\rho))$ in N

proof

According to the previous theorem, if $\rho$is sufficiently small and $\lambda\rightarrow\lambda^*$, $\mu\rightarrow\mu^*$ then x(λ,ρ)$\rightarrow$ x* and x(μ^*,ρ) $\rightarrow$ x*, however since λ^* is unknown the condition $\lambda\rightarrow\lambda^*$, $\mu\rightarrow\mu^*$cannot be enforced directly. Instead, theaugmented Lagrangian method updates λ using the results of the unconstrained minimization$\lambda \leftarrow \lambda_+(\lambda,\rho)$ $and\mu \leftarrow \mu_+(\mu,\rho)$ It is necessary to prove, then, that updating λ , μ in this manner produces a

sequence of Lagrange multiplier estimates converging to λ^* , μ^* Since λ_+ , $\mu_+ \square \square$ is a continuously differentiable function of λ , μ and $\lambda_+(\lambda^*, \rho) = \lambda^*$, $\mu_+(\mu^*, \rho) = \mu^*$, we can write

$$\lambda_+(\lambda, \rho) = \lambda^* + \int_0^1 \nabla\lambda_+(\lambda^* + t(\lambda - \lambda^*), \rho)^T(\lambda - \lambda^*)\,dt$$

Using the triangle inequality for integrals if follows

$$\|\lambda_+(\lambda, \rho) - \lambda^*\| \leq \int_0^1 \|\nabla\lambda_+(\lambda^* + t(\lambda - \lambda^*), \rho)^T\|\,\|\lambda - \lambda^*\|dt$$

$$\leq c(\rho)\|\lambda - \lambda^*\|$$

…(23)

Where $c(\rho)$ is an upper bounded for $\|\nabla\lambda_+(., \rho)^T\|$ Similarly

$$\mu_+(\mu, \rho) = \mu^* \int_0^1 \nabla\mu_+(\mu^* + t(\mu - \mu^*), \rho)^T(\mu - \mu^*)dt$$

$$\|\mu_+(\mu, \rho) - \mu^*\| \leq \int_0^1 \|\nabla\mu_+(\mu^* + t(\mu - \mu^*), \rho)^T\|\,\|\mu - \mu^*\|dt$$

$$\leq D(\rho)\,\|\mu - \mu^*\|$$

…(24)

Where $D(\rho)$ is an upper bounded for $\|\nabla\mu_+(., \rho)^T\|$ Similarly

$$x(\lambda, \rho) = x^* + \int_0^1 \nabla\mathbf{x}_+(\lambda^* + t(\lambda - \lambda^*), \rho)^T(\lambda - \lambda^*)\,dt$$

$$\|x(\lambda, \rho) - x^*\| \leq \int_0^1 \|\nabla\mathbf{x}_+(\lambda^* + t(\lambda - \lambda^*), \rho)^T\|\,\|\lambda - \lambda^*\|dt$$

$$\leq E(\rho)\,\|\lambda - \lambda^*\|$$

…(25)

Where $E(\rho)$ is an upper bounded for $\|\nabla\mathbf{x}_+(\lambda, \rho)^T\|$

$$x(\mu,\rho) = x^* + \int_0^1 \nabla \mathbf{x}_+ (\mu^* + t(\mu - \mu^*),\rho)^T (\mu - \mu^*)\, dt$$

$$\|x(\mu,\rho) - x^*\| \leq \int_0^1 \|\nabla \mathbf{x}_+ (\mu^* + t(\mu - \mu^*),\rho)^T\| \, \|\mu - \mu^*\| dt$$

$$\leq F(\rho)\, \|\mu - \mu^*\|$$
…(26)

Where $F(\rho)$ is an upper bounded for $\|\nabla\mathbf{x}_+(\mu,\rho)^T\|$

The function x,λ_+,μ_+ are defined by the equation.

$$\nabla f(x(\lambda,\mu,\rho)) + \lambda_+(\lambda,\rho)\nabla h(x) - \mu_+(\mu,\rho)\nabla g(x) = 0$$
$$\left(\lambda_+(\lambda,\rho) + \frac{\lambda}{n}\right) + \frac{2\rho^n}{n} h(x) = 0$$
$$n\mu_+(\mu,\rho)m + 2n\mu_+(\mu,\rho)\rho^n g(x) - 4\rho^{2n} = 0$$

Differentiating These equation respect to λ,μ and $Simplifying$ the results yields

$$\nabla^2 l\big(x(\lambda,\mu,\rho)\big),\lambda_+(\lambda,\rho),\mu_+(\mu,\rho)\big)\nabla x(\lambda,\mu)^T + \nabla\lambda_+(\lambda,\rho)^T\nabla h(x) - \nabla\mu_+(\mu,\rho)^T\nabla\mathbf{g}(x) = 0$$
…(27)

$$\frac{2\rho^n}{n}\nabla h(x)\nabla x(\lambda,\rho) + \nabla\lambda_+(\lambda,\rho) = 0$$
…(28)

$$\mathbf{2}n\mu_+(\mu,\rho)\rho^n\nabla g(x)\nabla(x)(\lambda,m) + n\mu\nabla\mu_+(\mu,\rho) + \mathbf{2}n\rho^n g(x)\nabla\mu_+(\mu,\rho) = \mathbf{0}$$ …(29)

$$J(x(\lambda,\mu,\rho),\lambda_+(\lambda,\rho),\mu_+(\mu,\rho)\lambda,\mu,\rho)\begin{bmatrix}\nabla x(\lambda,\mu)^T \\ \nabla\lambda_+(\lambda,\rho)^T \\ \nabla\mu_+(\mu,\rho)^T\end{bmatrix} = \begin{bmatrix}\mathbf{0} \\ \mathbf{0} \\ \mathbf{0}\end{bmatrix} \qquad …(30)$$

Since $J(x(\lambda,\mu,\rho),\lambda_+(\lambda,\mu),\lambda,\mu,\rho) \to J(x^*,\lambda^*,\mu^*,\lambda^*,\mu^*,\rho)$ as $\lambda \to \lambda^*$, $\mu \to \mu^*$ it follows that

$\|J(x(\lambda,\mu,\rho),\lambda_+(\lambda,\rho),\mu_+(\mu,\rho)\lambda,\mu,\rho)^{-1}\|$ is bounded above for all λ,μ sufficiently close to λ^*,μ^* therefore from

$$\begin{bmatrix} \nabla x(\lambda,\mu)^T \\ \nabla \lambda_+(\lambda,\rho)^T \\ \nabla \mu_+(\mu,\rho)^T \end{bmatrix} = \rho J(x(\lambda,\mu,\rho),\lambda_+(\lambda,\rho),\mu_+(\mu,\rho)\lambda,\mu,\rho)^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

…(31)

we can deduce That There exist $\rho^\wedge > 0$ and $M > 0$ such That for all $\rho \epsilon (0,\rho^\wedge)$

$$\|\nabla x(\lambda,\mu)^T\| \le \rho M \qquad , \quad \|\nabla \lambda_+(\lambda,\rho)^T\| \le \rho M$$

$$\|\nabla x(\mu,\rho)^T\| \le \rho M \qquad , \quad \|\nabla \mu_+(\mu,\rho)^T\| \le \rho M$$

Using ρM in place of C(P) , D(P) , E(P) , F(P)

Above , we obtain

$$\|\lambda_+(\lambda,\rho) - \lambda^*\| \le \rho M \|\lambda - \lambda^*\|$$

$$\|x(\lambda,\rho) - x^*\| \le \rho M \|\lambda - \lambda^*\|$$

$$\|\mu_+(\mu,\rho) - \mu^*\| \le \rho M \|\mu - \mu^*\|$$

$$\|x(\mu,\rho) - x^*\| \le \rho M \|\mu - \mu^*\|$$

For all $\rho \in (0,\rho)$

7 . Results and Conclusion

Several standard non-linear constrained test functions were minimized to compare the new algorithms with standard algorithm see (Appendix,B). with $1 \le m \le 4$ and $1 \le g_i(x) \le 4$. Is considered as the comparative performance of the following algorithm.

1- Mixed Equality–Inequality-Constrained Problems of the augmentd Lagrangian method (MAXAUG)

2- the Powell -Hestenes –Rockafellar of the augmented Lagrangian method (PHRAUG)

3- New Modified Barrier Augmented Lagrangian Method(MBAUG).

We denoted Mixed Equality–Inequality-Constrained Problems of the augmented Lagrangian method (MAXAUG),the Powell–Hestenes

Rockafellar of the augmented Lagrangian method (PHRAUG) , New Modified Barrier Augmented Lagrangian Method(MBAUG) .

All the results are obtained using (Laptop) . All programs are written in visual FORTRAN language and for all cases the stopping criterion taken to be $\|x_i - x_{i-1}\| < \delta$, $\delta = 10^{-5}$

All the algorithms in this paper use the same ELS strategy which is the quadratic interpolation technique directly adapted from (Bunday ,1984) .

The comparative performance for all of these algorithms are evaluated by considering NOF, NOI, NOG and NOC, where NOF is the number of function evaluation and NOI is the number of iteration and NOG is the number of gradient evaluation and NOC number of constrained evaluation.

In table (1) we have compared of three algorithms (MAXAUG) , (PHRAUG) , (MBAUG) .

Table (1)

Comparison of (MAXAUG) , (PHRAUG) , (MBAUG) .

NO.	MAX AUG. NOF(NOG)NOI(NOC)	PHRAUG. NOF(NOG)NOI(NOC)	MBAUG. NOF(NOG)NOI(NOC)
1	1544(150)2(1)	259(66)3(3)	23(21)2(1)
2	357(28)2(1)	72(7)2(1)	103(9)5(9)
3	144(3)2(1)	144(3)2(1)	9(4)2(1)
4	213(13)2(1)	167(5)2(1)	96(6)2(1)
5	137(3)2(1)	147(3)2(1)	106(8)2(1)

6	47(4)2(1)	41(3)2(1)	146(9)2(1)
7	36(3)2(1)	76(8)2(1)	6(3)2(1)
8	119(2)2(1)	141(2)2(1)	95(7)5(9)
9	46(5)5(9)	2393(58)2(1)	100(9)5(9)
10	41(2)2(1)	289(7)5(1)	341(1)1(1)
TO.	2684(213)23(18)	3729(162)24(12)	1025(77)28(34)

Fig(1)
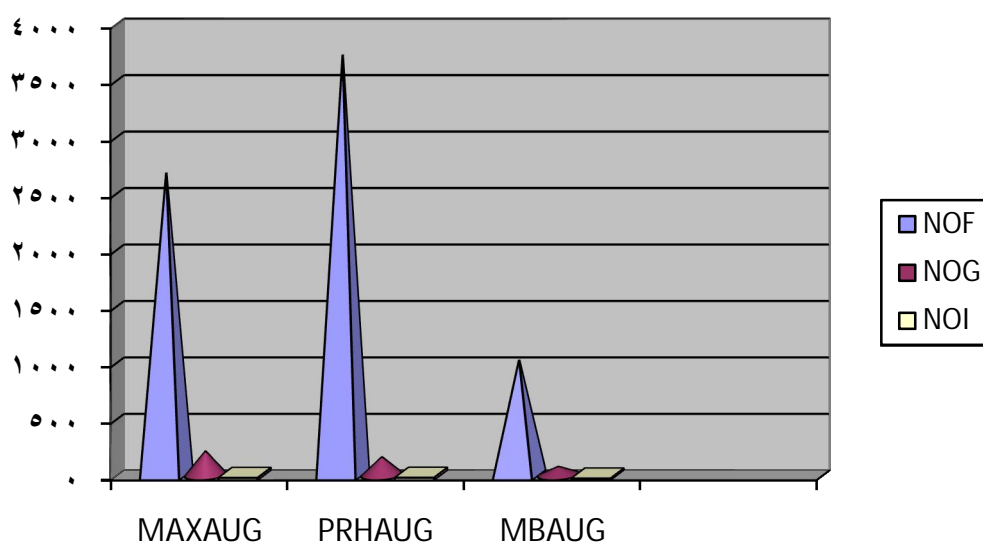
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<div dir="rtl">

(Appendix B) الملحق

دوال الاختبار لامثلية المقيدة

</div>

Appendix

1. min $f(x) = x_1^2 - x_1 x_2 + x_2^2$

   s.t

$x^2_1 + x^2_2 - 4$
$- 2x_1 - x_2 + 2$

$x = [0.02 , 0.04]$

2. min $f(x) = (x_1 - 2)^2 + .25\, x_2^2$

   s.t

$2x_1 + 3x_2 - 4$
$- x_1 + 3.5x_2 + 1$

$x = [3,5]$

3. min $f(x) = x_1^2 + x_2^2$

   s.t

$x_1 + 2x_2 - 4$

$-x_1^2 - x_2^2 + 5$

x_1

x_2

$x = [.9, 1.3]$

4. min $\quad f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$

  s.t

$x_1 - 2x_2 + 1$

$-\dfrac{x_1^2}{4} - x_2^2 + 1$

$x = [.7, .7]$

5. min $\quad f(x) = (x_1 - 3)^2 + (x_2 - 2)^2$

  s.t

$x_1 + 2x_2 - 4$

$-x_1^2 - x_2^2 + 5$

x_1

x_2

$x = [0.2, 1]$

6. min $\quad f(x) = (x_1 - 1)^2 + 2(x_2 - 3)^2 + (x_3 + 1)^2$

  s.t

$x_1^2 + 4x_3^2 - 6$

x_1

x_2

x_3

$x = [3, 3, 3]$

7. min $\quad f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$

  s.t

$$-x_1^2 + x_2$$
$$x_1 + x_2 - 2$$
$$x = [2,2]$$

8. min $f(x) = (x_1 - 2)^2 + (x_2 - 4)^2$

s.t

$$2x_1^2 + x_2 - 34$$
$$2x_1 + 3x_2 - 18$$
$$x_i > 0$$

9. min $f = x_1 x_2$

s.t

$$25 - x_1^2 - x_2^2$$
$$x_1 + x_2$$
$$x_i > 0$$

10. min $f = -2x_1 - x_2$

s.t

$$x_1^2 + x_2^2 \le 25$$
$$x_1^2 - x_2^2 \le 27$$
$$x_i > 0$$