Iraqi Journal of Statistical Sciences

http://stats.uomosul.edu.iq

Comparison of Estimation Methods for the Parameters of the Frechet Distribution - using Simulation

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Article information

Article history:
Received October 15,2024
Revised:April 1,2025
Accepted :April 20,2025
Available June 1, 2025

Keywords:
Probability Distributions, Frechet Distribution, Estimation Methods, Simulation

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Abstract

Probability distributions are mathematical functions that describe the likelihood of different outcomes in random process estimates for the scale parameters and the shape parameter according to the type of data that can determine the appropriate probability distribution. In this paper, an experimental study is presented to compare a number of estimation methods for the parameters of the Frechet distribution, which is one of the most important probability distributions in the fields of determining failure times. The estimation Methods are (Maximum Likelihood, Moments and Bayesian methods) were adopted. Through the simulation method, the comparison process was carried out, where the experimental samples were determined (n = 15, 25, 50, 75, 100) with the assumption of four default values for each of the shape parameter (λ =1.1, 1.5, 2, 2.5) and the scale parameter (θ=1.4, 1.8, 2.3, 3). Through this method, the paper was able to determine the appropriate method for estimation by adopting the Mean square error criterion. The experimental results showed the superiority of the Bayes method. Then the method of Maximum likelihood.

DOI 10.33899/iqjoss.2025.187758, ©Authors, 2025, College of Computer Science and Mathematics, University of Mosul.

1. Introduction

Probability distributions have received great attention in addressing many problems through estimation and inference processes about the accuracy of the estimated parameters. The interest in probability distributions began to be reflected in the treatment of the associated analysis of the reliability of systems and the neutralization of failure times. Therefore, this paper adopted a distribution through which we can obtain the best estimate for the parameter of scale and shape (θ, λ) for the distribution of Frechet as a special case of the general distribution of extreme values. The multiplicity of estimation methods puts the researcher in the path of verifying their preference. Therefore, there was great interest in the areas of determining the appropriate methods of estimation, which emerged through scientific contributions in this field, as Khader and others(2009) presented a comparative study with the adoption of a number of methods with the aim of estimating the parameter of the exponential distribution to estimate the parameter of failure times, and they were able to reach that the best method is the greatest place method (Khader, Hassoun, & Hussein, 2009). Catelani et al. (2016) adopted methods of estimating parameters to determine the best distribution of a set of data, where the parameters were estimated by adopting the methods of the least squares and the Maximum likelihood by adopting failure time data to determine the distribution that best fits the data (Catelani, Ciani, Guidi, & Venzi, 2016). Tablada and Cordeiro (2016) proposed a modified distribution for the Frigate distribution by modelling the distribution with three parameters, to expand the Frechet distribution. This was done by applying the Lambert function to achieve some characteristics of the modified distribution. The results showed that the distribution has flexibility while proving its applied importance (Tablada & Cordeiro, 2017). Al Wakeel and Laibi (2019) They addressed the problem of violating the estimation conditions when the data is contaminated with outliers by using robust methods to reach robust estimators that have characteristics when the assumed distribution deviates from the normal distribution due to the presence of anomalies in its values. The paper used the power function distribution because of its flexibility and ability to model data in various sectors. (Al Wakeel & Laibi, 2019). Ciani and Guidi in (2019) focused on distributions through the paper on the exponential distributions and Weibull distributions and comparing them through the basic failure distributions. The paper focused on the fixed failure rate of the exponential distribution and the time-dependent behavior of Weibull. The paper was able to find that the LSE method is analytical and gives good results in the case of a small number of samples, while MLE was the most accurate and appropriate technique in the case of large samples (Ciani & Guidi, 2019).Ramos and others studied the problem of estimating the parameters of the Frechet distribution through two iterative and bizarre methodologies by adopting a number of estimation methods and five sets of real data related to the minimum flow of water on the Piracicaba River in Brazil to clarify the applicability of these methods. The results showed that the Frechet distribution achieved a good advantage in the estimation process (Ramos, Louzada, Ramos, & Dey, 2020). Khdair and Aboudi in (2022) proposed a new distribution of the exponential power function by building a distribution with four parameters, relying on the addition of a new shape parameter for the distribution function, relying on the method of exponential expansion, which is the basis for obtaining a distribution belonging to the exponential family. To determine the features of the model, the Greatest Possibility Method Mle and the LSE method were adopted. The results showed that there is convergence between the two methods (Khdair & Aboudi, 2022). Noaman et al. (2023), the mayor of the paper, approved the distribution of Weibull and Frechet as an appropriate distribution of the amount of rain achieved in some Iraqi governorates. The paper was able to challenge the appropriate distribution for each of the governorates and the method of estimation was the Maximum likelihood method according to the evaluation criteria (Noaman, Abdul Ameer, & Mohammed, 2023).

2. Material and Methodology

2.1. Frechet Distribution

The Frechet distribution is a special case of the general distribution of extreme values, which was defined by the French mathematical scientist Maurice Frechet (1927) and has been further transformed by the scientist Fisher & Tippet (1928) and later Gumbel (1958) and is known as the following probabilistic function (p.d.f) (Ramos, Louzada, Ramos, & Dey, 2020): -

$$f(t;\lambda,\theta)=\begin{cases}\lambda\theta^{\lambda}t^{-(\lambda+1)}e^{-(\frac{\theta}{t})^{\lambda}} & I(t)_{(0,\infty)} \\ 0 & \frac{w}{o}\end{cases} \tag{1}$$

Where:

λ: Represents the shape parameter

θ: represents the scale parameter

The following figure represents the distribution of Frechet with different values to the shape parameter and the scale parameter

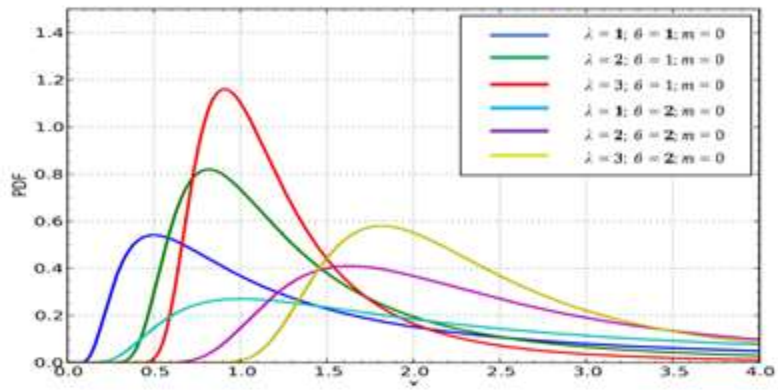

Figure (1) shows the two-parameter Frechet distribution curve for different values of the shape and scale parameters

It is noted from the figure that the probability function is decreasing

The cumulative distribution function can be obtained from:

$$F(t) = p(T \leq t) = \int_0^t f(u)du$$

$$= \boldsymbol{e^{-(\frac{\theta}{t})^{\lambda}} t > 0; \lambda > 0; \theta > 0} \qquad (\mathbf{2})$$

2.2. Estimation Methods

2.2.1 Maximum Likehood Method

The Maximum Likehood Method (Mle) function is one of the important methods in diagnosing the minimum random error limits for any probability distribution, which contributes to finding the optimal values for the parameters of the figure and measuring that distribution. We were in the process of estimating the parameters of the probability distribution Frechet This method will contribute to proving the characteristics of the distribution parameters as they are characterized by stability, that is, if they $\theta_{ML}, \lambda_{ML}$ are the estimates of the greatest potential of the two parameters, θ, λ they achieve the evidentiary requirements for the probability density function of distribution as follows (Alharbi & Hamad, 2024):

$$L(t_1, t_2, \ldots, t_n; \lambda, \theta) = \prod_{i=1}^{n} \lambda \theta^{\lambda} t^{-(\lambda+1)} e^{-(\frac{\theta}{t})^{\lambda}}$$

$$= \lambda^n (\theta^{\lambda})^n e^{-\sum_{i=1}^{n}(\frac{\theta}{t_i})^{\lambda}} \prod_{i=1}^{n} t_i^{-(\lambda+1)} \qquad (\mathbf{3})$$

For the purpose of estimating the potential function, it must be converted to the linear form by taking the natural logarithm of both sides of the equation (3), where we obtain:

$$LnL = nLn\lambda - n\lambda Ln\theta - (\lambda + 1)\sum_{i=1}^{n} ln\, \boldsymbol{t_i} - \sum_{i=1}^{n}\left(\frac{\theta}{t_i}\right)^{\lambda} \qquad (4)$$

In order to find the estimated values of the location, shape and scale parameter that make the possible function as great as possible, this is done by calculating the maximum limits of the function (4) as follows (Harter & Moore, 1965) (Al-Yasiri, 2007):

Equation (4) will be derived for (θ) as the shape parameter (λ) is known

$$\frac{\partial LnL}{\partial \theta} = \frac{n\lambda}{\hat{\theta}} - \lambda\hat{\theta}^{\lambda-1}\sum_{i=1}^{n}\left(\frac{1}{t_i}\right)^{\lambda} = 0$$

$$\frac{\partial LnL}{\partial \theta} = n\lambda - \lambda\hat{\theta}^{\lambda}\sum_{i=1}^{n}\left(\frac{1}{t_i}\right)^{\lambda} = 0$$

Therefore, the estimator of the greatest places of the scale parameter is

$$\hat{\theta}_{ML} = \left[\frac{n}{\sum_{i=1}^{n}\left(\frac{1}{t_i}\right)^{\lambda}}\right]^{\frac{1}{\lambda}} \tag{5}$$

2.2.2 Method of Moment Estimators (MOM)

The Moment estimation method is one of the commonly used methods in the field of parameter estimation, which was proposed by Bernaolli and Johan (1667-1748), as it is characterized by its simplicity and depends on equating the population moment μ_r with the sample moment m_r and finding an estimated formula for the parameters (Hansen, 1982) (Gove, 2003)**.**

$$m_r = \frac{\sum_{i=1}^{n} t_i^r}{n} \tag{6}$$

$$m_r = \mu_r = E(T^r).$$

In order to obtain the estimator, the general formula of community determination must first be derived.

$$E(T^r) = \int_0^{\infty} t^r.\lambda\theta^{\lambda}t^{-(\lambda+1)}\, e^{-(\frac{\theta}{t})^{\lambda}}dt \tag{7}$$

$$= \lambda\theta^{\lambda}\int_0^{\infty} t^r.t^{-(\lambda+1)}\, e^{-(\frac{\theta}{t})^{\lambda}}dt \tag{8}$$

Let

$$y = \frac{\theta}{t} \Rightarrow t = \frac{\theta}{y} \Rightarrow dt = \frac{\theta}{y^2}dy$$

After compensation in equation (8) and simplification, we find that

$$= \lambda\theta^r\int_0^{\infty}(\frac{1}{y})^{r-\lambda+1}.e^{-(y)^{\lambda}}dy \tag{9}$$

Let's make $letw = (y)^{\lambda} \Rightarrow w^{\frac{1}{\lambda}} = y \Rightarrow \frac{1}{\lambda}w^{\frac{1}{\lambda}-1} = dy$

Then:

$$= \theta^r\int_0^{\infty}(\frac{1}{w^{\frac{1}{\lambda}}})^{r-\lambda+1}w^{\frac{1}{\lambda}-1}.e^{-(w)}dw$$

$$E(t^r) = \theta^r\int_0^{\infty}(W)^{-\frac{r}{\lambda}}.e^{-(w)}dw \Rightarrow = \theta^r\int_0^{\infty}(W)^{(1-\frac{r}{\lambda})-1}.e^{-(w)}dw$$

So, the intentions of the community are

$$E(t^r) = \theta^r\Gamma\left(1-\frac{r}{\lambda}\right) \tag{10}$$

So the torque estimator for the scale parameter

$$\hat{\theta}_{mom} = \frac{\bar{x}}{\Gamma(1-\frac{1}{\lambda})} \tag{11}$$

2.2.3 Bayesian Methods Estimation

The Bayesian approach is one of the mathematical methods that look at determining the prior distribution of the parameters by assuming them to be random variables, relying on experience and the nature of the data. According to the Bayesian theory of estimation, we can formulate the following formula (Muehlemann, Zhou, Mukherjee, Hossain, & Roychoudhury, 2023):

$$h(\theta|t) = \frac{P(t|\theta)P(\theta)}{p(t)} \tag{12}$$

The loss function has an impact in determining the Bayes estimator and represents the loss function as a result of making a decision. There are several types of loss function, the most common and used of which is the quadratic loss function, which will be adopted in the research, which is as follows:

$$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2 \tag{13}$$

2.3 Non-Informative Priority Potential Density P.d.f.

When there is insufficient information about the parameters to be estimated or not available at all, the (Jeffery) method is followed, which includes two rules in the Prior function choices of the unknown parameters. The first is in the case if the parameter θ field is an infinite field (-∞,∞). The pre-functional probability distribution is a regular distribution. If the parameter field is any positive field (0,∞), its probability distribution is a regular logarithmic distribution and the function is improper, that is, its integration over its field is not equal to one, but when it is combined with the possibility function of sample observations, we get an appropriate function to estimate the unknown parameters (Grzenda, 2016).

$$\prod(\theta) \propto \sqrt{Det[I(\theta)]}, \theta > 0 \tag{14}$$

Accordingly, an prior distribution of the estimator of the scale parameter θ was proposed as follows:

$$\prod(\theta) = k(\theta^\lambda)^{B-1}. e^{-B\theta^\lambda}, \theta > 0 \tag{15}$$

Thus, the posterior probability density function will be (Posterior p.d.f)

$$h(\theta|t) = \frac{\prod_{i=1}^{n} f(t_i|\theta) \prod(\theta)}{\int_0^\infty f(t_i|\theta)\prod(\theta)d\theta} \tag{16}$$

This is banned by the local DNO.

$$\int_0^\infty f(t_i|\theta)\prod(\theta)d\theta = \int_0^\infty \lambda^n(\theta^\lambda)^n \prod t_i^{-(\lambda+1)}\, e^{-\sum(\frac{\theta}{ti})^\lambda} k(\theta^\lambda)^{B-1}. e^{-B\theta^\lambda} d\theta$$

$$= k\lambda^n \prod t_i^{-(\lambda+1)} \int_0^\infty (\theta^\lambda)^n\, (\theta^\lambda)^{B-1} e^{-\theta^\lambda(B+\sum(\frac{1}{ti})^\lambda)} d\theta$$

$$= k\lambda^n \prod t_i^{-(\lambda+1)} \int_0^\infty (\theta^\lambda)^{n+B-1}\, e^{-\theta^\lambda(B+\sum(\frac{1}{ti})^\lambda)} d\theta \tag{17}$$

$$let \quad y = (\theta)^\lambda \Rightarrow y^{\frac{1}{\lambda}} = \theta \Rightarrow \frac{1}{\lambda} y^{\frac{1}{\lambda}-1} dy = d\theta$$

We don't get

$$= \frac{k\lambda^n \prod t_i^{-(\lambda+1)}}{\lambda} \int_0^\infty (y)^{n+B-1+\frac{1}{\lambda}-1}\, e^{-y(B+\sum(\frac{1}{ti})^\lambda)} dy$$

$$= k\lambda^{n-1} \prod t_i^{-(\lambda+1)} \int_0^\infty (y)^{n+B-1+\frac{1}{\lambda}-2}\, e^{-y(B+\sum(\frac{1}{ti})^\lambda)} \frac{1}{\lambda} y^{\frac{1}{\lambda}-1} dy \tag{18}$$

We assume $T = (B + \sum(\frac{1}{ti})^\lambda)$

After substitution, the equation becomes (15)

$$= k\lambda^{n-1}\prod t_i^{-(\lambda+1)}\int_0^{\infty} y^{(n+B-1+\frac{1}{\lambda}-2)}\, e^{-yT}\,\frac{1}{\lambda} y^{\frac{1}{\lambda}-1} dy$$

$$= k\lambda^{n-1}\prod t_i^{-(\lambda+1)}\,\frac{\Gamma(n+B+\frac{1}{\lambda}-1)}{T^{n+B+\frac{1}{\lambda}-1}}$$

$$h(\theta|t) = \frac{\lambda\lambda^n(\theta^\lambda)^n \prod t^{-(\lambda+1)} e^{-\Sigma\left(\frac{\theta}{t_i}\right)^\lambda} k(\theta^\lambda)^{B-1}.\, e^{-B\theta^\lambda}}{k\lambda^{n-1}\prod t^{-(\lambda+1)}\,\frac{\Gamma(n+B+\frac{1}{\lambda}-1)}{T^{n+B+\frac{1}{\lambda}-1}}}$$

$$h(\theta|x) = \frac{\lambda(\theta^\lambda)^{n+B-1} e^{-\theta^\lambda(B+\Sigma\left(\frac{1}{t_i}\right)^\lambda)}}{\frac{\Gamma(n+B+\frac{1}{\lambda}-1)}{\left(B+\Sigma\left(\frac{1}{t_i}\right)^\lambda)\right)^{n+B+\frac{1}{\lambda}-1}}} \tag{19}$$

On this basis, the subsequent distribution of the teacher is

$$h(\theta|t) = \frac{\lambda(\theta^\lambda)^{n+B-1}\left(B+\Sigma\left(\frac{1}{t_i}\right)^\lambda)\right)^{n+B+\frac{1}{\lambda}-1} e^{-\theta^\lambda(B+\Sigma\left(\frac{1}{t_i}\right)^\lambda)}}{\Gamma(n+B+\frac{1}{\lambda}-1)} \tag{20}$$

After obtaining the post-distribution function of the parameter, the loss function θ is calculated: -

$$\therefore \hat{\theta}_{Bayes} = E(\theta|t) \tag{21}$$

To find the estimator, we extract the conditional mean E(θ|x), which is equal to

$$E(\theta|t) = \int \theta g(\theta|t)\, d\theta \tag{22}$$

$$= \int_0^{\infty} \theta\,\frac{\lambda\left(\theta^\lambda\right)^{n+B-1}(B+\Sigma\left(\frac{1}{t_i}\right)^\lambda)^{n+B+\frac{1}{\lambda}-1} e^{-\theta^\lambda(B+\Sigma\left(\frac{1}{t_i}\right)^\lambda)}}{\Gamma(n+B+\frac{1}{\lambda}-1)} d\theta\,;\, let\ T = B+\sum\left(\frac{1}{t_i}\right)^\lambda$$

$$= \int_0^{\infty} \theta\,\frac{\lambda\left(\theta^\lambda\right)^{n+B-1} T^{n+B+\frac{1}{\lambda}-1} e^{-\theta^\lambda(B+\Sigma\left(\frac{1}{t_i}\right)^\lambda)}}{\Gamma(n+B+\frac{1}{\lambda}-1)}\, d\theta$$

$$= \frac{\lambda T^{n+B+\frac{1}{\lambda}-1}}{\Gamma(n+B+\frac{1}{\lambda}-1)}\int_0^{\infty}\theta\left(\theta^\lambda\right)^{n+B-1} e^{-\theta^\lambda T}\, d\theta$$

We make the following transfer

$$let\ w = (\theta)^\lambda \Rightarrow w^{\frac{1}{\lambda}} = \theta \Rightarrow \frac{1}{\lambda} w^{\frac{1}{\lambda}-1} dw = d\theta$$

And by compensating her, we get

$$= \frac{T^{n+B+\frac{1}{\lambda}-1}}{\Gamma\left(n+B+\frac{1}{\lambda}-1\right)} \int_0^\infty w^{\frac{2}{\lambda}-1}\ (w)^{n+B-1}\ e^{-wT}\ dw$$

$$= \frac{T^{n+B+\frac{1}{\lambda}-1}}{\Gamma\left(n+B+\frac{1}{\lambda}-1\right)} \int_0^\infty w^{n+B+\frac{2}{\lambda}-2}\ e^{-wT}\ dw$$

$$= \frac{T^{n+B+\frac{1}{\lambda}-1}}{\Gamma(n+B+\frac{1}{\lambda}-1)} \left[\frac{\Gamma(n+B+\frac{2}{\lambda}-1)}{T^{n+B+\frac{2}{\lambda}-1}}\right] \tag{23}$$

Therefore, the new base proposal under the quadratic loss function is the conditional average and is equal to

$$E(\theta|t) = \hat{\theta}_b = \frac{\Gamma(n+B+\frac{2}{\lambda}-1)}{\Gamma(n+B+\frac{1}{\lambda}-1)}\left(\frac{1}{T^{\frac{1}{\lambda}}}\right) \tag{24}$$

3 Experiential Aspect

The simulation method is considered one of the mathematical methods to follow the programmatic path to solve complex problems, especially the problems that arise during the design of inspection plans. Accordingly, the simulation scheme was built using the following algorithm MATLAB version 23 was adopted for the purpose of implementing the simulation:

1- Define default sample sizes, n = 15, 25, 50, 75 and 100.

2- Defining the default values of the parameters Four default values were selected for the shape parameter λ and four default values for the parameter θ, and as shown in the following table:

Table (1)
Default values of shape parameter λ and scale parameter θ for Frechet **distribution**

2.5	2	1.5	1.1	λ
3	2.3	1.8	1.4	θ

3- Determine the frequency of each experiment (1000) times to obtain accuracy and homogeneity in estimating the parameters.

4- Data generation, where the Inverse Transform method was used on random observations with a Frechet distribution resulting from random observations generated from one community of regular distribution (0,1) for the purpose of obtaining observations with a Frechet distribution, through the use of the cumulative distribution function (C.D.F) that describes growth C:

U= F(t)
T=F^{-1}(u)

5- Estimate Frechet Distribution Parameters

6- Compare the estimation methods by adopting the average error squares (MSE) shown in the formulas below:

$$MSE(\hat{\theta}) = \frac{1}{L}\sum_{i=1}^{L}(\hat{\theta}_i - \theta)^2 \quad (25)$$

When implementing the simulation stages, the following results were reached:

Table (2) Comparison of methods of estimation according to the MSE standard when adding the values of λ and $\boldsymbol{\theta = 1.4}$

N	of Disbursement	λ / θ	1.1	1.5	2	2.5
15	Mom	1.4	1.748	1.4654	0.5687	0.1012
	Bayes		0.0643	0.0413	0.0243	0.0188
	Mle		0.1835	0.0705	0.039	0.0241
Test method			Bayes	Bayes	Bayes	Bayes
25	Mom	1.4	2.662	1.3098	0.2294	0.0663
	Bayes		0.0441	0.0271	0.0162	0.0113
	Mle		0.0803	0.0376	0.0217	0.0133
Test method			Bayes	Bayes	Bayes	Bayes
50	Mom	1.4	3.7361	2.2656	0.1793	0.0359
	Bayes		0.0272	0.015	0.0086	0.006
	Mle		0.0364	0.0175	0.0101	0.0067
Test method			Bayes	Bayes	Bayes	Bayes
100	Mom	1.4	2.4249	4.8149	0.0639	0.0183
	Bayes		0.0142	0.0079	0.0047	0.0032
	Mle		0.0165	0.0087	0.0051	0.0033
Test method			Bayes	Bayes	Bayes	Bayes

It is clear from the results of the table (2) that the preference was for the Bayes method, where the lowest error rate was recorded for the estimate at all parameter values λ and $\theta = 1.4$ for all sizes of samples assumed compared to the two methods of moments and the greatest place. It was also noted that the value of the average error squares **decreases when the sample size increases.**

Table (3) Comparison of methods of estimation according to the MSE standard when adding the values of λ and $\theta = 1.8$

N	Method	λ / θ	1.1	1.5	2	2.5
15	Mom	1.8	6.0935	6.0424	0.2662	0.1699
	Bayes		0.1145	0.0814	0.0682	0.0735
	Mle		0.2466	0.1305	0.0628	0.0378
Best Method			Bayes	Bayes	Mle	Mle
25	Mom	1.8	5.6285	7.7043	0.2411	0.3092
	Bayes		0.0806	0.046	0.0306	0.0368
	Mle		0.1215	0.0639	0.0356	0.0226
BEST			Bayes	Bayes	Bayes	Mle
50	Mom	1.8	3.7864	2.0141	0.2757	0.0563
	Bayes		0.0424	0.0272	0.0161	0.0151
	Mle		0.056	0.0324	0.0166	0.011
BEST			Bayes	Bayes	Bayes	Mle
100	Mom	1.8	5.6486	0.5803	0.0816	0.0368
	Bayes		0.0246	0.0139	0.0084	0.0064
	Mle		0.0287	0.0157	0.0082	0.005
BEST			Bayes	Bayes	Mle	Mle

It is clear from the results of the table (3) that the preference was for the Bayes method and then the Mle method, where the Bayes method recorded the lowest error rate for estimation when $\boldsymbol{\lambda = 1.1, 1.5}$; $\boldsymbol{\theta = 1.8}$ and for all assumed sample sizes, while the Greatest Possibility method achieved the lowest MSE when $\boldsymbol{\lambda = 2.5}$; $\boldsymbol{\theta = 1.8}$ and for all sample sizes. It was also noted that the value of the average error squares **decreases when the sample size increases.**

Table (4) Comparison of methods of estimation according to the MSE standard when adding the values of λ and $\theta = 2.3$

n	Method	λ / θ	1.1	1.5	2	2.5
15	Mom	2.3	5.9098	4.1123	0.791	0.3156
	Bayes		0.3038	0.1504	0.2233	0.2851

	Mle		0.3588	0.1824	0.1003	0.0635
BEST			Bayes	Bayes	Mle	Mle
25	Mom	2.3	8.8262	2.012	0.4953	0.1447
	Bayes		0.1806	0.101	0.1171	0.1518
	Mle		0.2358	0.1139	0.0616	0.0365
BEST			Bayes	Bayes	Mle	Mle
50	Mom	2.3	5.4162	1.1407	0.2314	0.1207
	Bayes		0.0886	0.045	0.043	0.055
	Mle		0.0964	0.0463	0.0286	0.0181
BEST			Bayes	Bayes	Mle	Mle
100	Mom	2.3	4.4884	2.0862	0.1783	0.0517
	Bayes		0.0471	0.0239	0.018	0.0194
	Mle		0.0509	0.0242	0.0132	0.0081
BEST			Bayes	Bayes	Mle	Mle

It is clear from the results of the table (4) that the preference was for the proposed Bayes method when $\boldsymbol{\lambda = 1.1, 1.5}$; $\boldsymbol{\theta = 2.3}$ and for all the assumed sample sizes, while the preference was for the Greatest Possibility method, where the lowest MSE was recorded when $\boldsymbol{\lambda = 2, 2.5}$; $\boldsymbol{\theta = 2.3}$ and for all sample sizes. It was also noted that the value of the average error squares **decreases when the sample size increases.**

Table (5) Comparison of methods of estimation according to the MSE standard when adding the values of λ and θ = 3

N	method	λ / θ	1.1	1.5	2	2.5
15	Mom	3	6.5624	2.8858	1.3025	0.4998
	Bayes		0.666	1.935	0.7923	1.0568
	Mle		0.7038	3.0803	0.1886	0.1093
BEST			Bayes	Bayes	Mle	Mle
25	Mom	3	9.2145	3.2104	1.0801	0.3303
	Bayes		0.2012	0.1063	0.4371	0.6288
	Mle		0.3744	0.1615	0.0967	0.0678
BEST`			Bayes	Bayes	Mle	Bayes
50	Mom	3	7.7931	4.322	1.7686	0.1706

	Bayes		0.2205	0.02	0.1696	0.2584
	Mle		0.169	0.0835	0.0451	0.0321
BEST			Mle	Bayes	Mle	Mle
100	Mom	3	6.5006	1.8207	0.219	0.1142
	Bayes		0.0757	0.0535	0.061	0.0917
	Mle		0.0785	0.0405	0.0225	0.0139
BEST			Bayes	Mle	Mle	Mle

It is clear from the results of the table (5) that the preference was for the Bayes method $\boldsymbol{\lambda = 1.1, 1.5}$; $\boldsymbol{\theta = 3}$ when at the sample sizes **(n = 15, 25, 50)**, while the preference was for the Greatest Possibility method $\boldsymbol{\lambda = 2, 2.5}$; $\boldsymbol{\theta = 3}$ when at the sample sizes **(n = 15, 25, 50)**, while the preference was for the Maximum likelihood method $\boldsymbol{\lambda = 1.5, 2, 2.5}$; $\boldsymbol{\theta = 3}$ when at the sample size (n = 100). It was also noted that the value of the average error squares is decreasing **when increasing** the **sample size.**

The experimental results concluded that the ratio of preference was due to the proposed Bayes method at most sample sizes and then the Greatest Possibility method.

Conclusions

1-The paper concluded that the Bayes method is the best method of estimation and then comes the method of the greatest possible.

2-The experimental Bayesian method achieved the lowest MSE at all sample sizes at all default values of the shape parameter (λ =1.1, 1.5, 2, 2.5) and at the default value of the scale parameter ($\theta = 1.4$).

3-It was noted when increasing the default values of the scale parameter ($\theta = 1.8, 2.3, 3$) at all the default values of the shape parameter (λ =1.1, 1.5, 2, 2.5) that the preference fluctuates between the experimental Bayesian method and the maximum likelihood method.

Recommendations

1- The paper recommends the adoption of other estimation methods with the adoption of smart methods for the purpose of comparing them in determining the appropriate method.
2- Using methods with fuzzy logic in determining the appropriate method of estimation.
3- Adopting mixed or compound distributions for the purpose of comparing the methods of estimation in light of the non-linearity of the data.
4- Applying this type of studies to practical reality in engineering and industrial fields.

Acknowledgment

The authors are very grateful to the University of Mosul, and College of Computer and Mathematical Science, which helped improve this work's quality.

Conflict of interest

The author has no conflict of interest.

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مقارنة طرائق التقدير لمعالم توزيع فرجت – باستعمال المحاكاة.

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الخلاصة: تعتبر التوزيعات الاحتمالية احد الاساليب الرياضية التي تستهدف الحصول على أفضل المقدرات لمعلمتي القياس (θ) ومعلمة الشكل (λ) طبقاً لنوعية البيانات التي يمكن ان يحدد التوزيع الاحتمالي الملائم حيث في هذه الورقة تقديم دراسة تجريبية للمقارنة بين عدد من طرائق التقدير لمعالم توزيع فرجت الذي يعد احد اهم التوزيعات الاحتمالية في مجالات تحديد ازمنة الفشـــل، حيث تم اعتماد طرائق التقدير (الامكان الاعظم، العزوم و طريقة بيز) ومن خلال اســـلوب المحاكاة تم اجراء عملية المقارنة حيث حددت العينات التجريبية ($n = 15, 25, 50, 75, 100$) مع افراض اربع قيم افتراضـــية لكل من معلمة الشـــكل (λ=1.1, 1.5, 2, 2.5) ومعلمة القياس ($\theta = 1.4, 1.8, 2.3, 3$) وتمكنت الورقة من خلال هذا الاســـلوب التوصـــل الى تحديد الطريقة المناســـبة للتقدير باعتماد معيار (MSE) وقد بينت النتائج التجريبة تفوق طريقة بيز ومن ثم طريقة الامكان الاعظم.

الكلمات المفتاحية: التوزيعات الاحتمالية، توزيع فرجت، طرائق التقدير ، اسلوب المحاكاة.