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# Bayesian Time Series Modelling with Wavelet Analysis for Forecasting Monthly Inflation

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**Abstract**

This article treats data noise and outliers in Bayesian ARIMA models through wavelet analysis. Apply the discrete wavelet transformation using Daubechies and Symlets wavelets for orders 10 and 15 to decompose the data of Bayesian ARIMA models into their frequency components. Threshold the wavelet coefficients using a method like soft thresholding, with the threshold selected via Stein's unbiased risk estimate and soft rule. Simulation experiments were used with real data representing the monthly inflation in the Kurdistan Region of Iraq (2009-2024) with a forecast for the next ten months. The proposed wavelet-based Bayesian ARIMA method provides a robust framework for handling noisy time series data and offers significant improvements over classical methods, making it an appealing choice for practical applications in time series forecasting, particularly when dealing with outliers and noise.

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## 1. Introduction

One of the economic issues worthy of attention is the forecasting of monthly inflation, which is one of the basic problems that affect financial decisions and monetary policies, as it relies on traditional and classical methods of analyzing time series, including ARIMA models and exponential smoothing, which rely on static hypotheses that may not be appropriate in the presence of structural changes or complex dynamic characteristics (. Therefore, there is an urgent need for more flexible and accurate models in the forecasting process, such as time series modelling based on Bayesian theory after combining it with Wave analysis, to improve the accuracy of forecasts. (Box et al., 2016; Heyam et al.,2025) The use of wavelet analysis to show time-varying patterns within a time series makes it a powerful and effective tool for analyzing monthly inflation. The role of wavelets in breaking up the time series into its various components is highlighted, so that it allows dealing with structural and seasonal changes that may be unclear when using traditional methods by relying on Bayesian theory using probability distributions to extract future, predictive values, which allows dealing with uncertainty more flexibly than classical methods. Dynamic Bayes models can integrate previous information about inflation and gradually improve the estimation of parameters, which makes them suitable for forecasting under the instability of economic data. (Hayawi & Alsharabi, 2022; Muzahem & Hayawi, 2023)

This article treats data noise and outliers in Bayesian ARIMA models through wavelet analysis. Apply the discrete wavelet transformation using Daubechies and Symlets wavelets for orders 10 and 15 to decompose the data of Bayesian ARIMA models into their frequency components.

## 2. Methods

### 2.1. ARIMA

ARIMA models One of the important tools in analyzing and predicting time series are ARIMA model, which is called the model of self-integration and mean regression, which is one of the most famous Time Series models, which is characterized and characterized by its ability to clarify trends and self-reliance in time series data, (Hayawi et al., 2021) as it is suitable for predicting changes that occur in various phenomena, especially data characterized by periodic trends, and these models give more flexibility when dealing with Wave analysis and integrating them in an estimation method that relies on Bayesian theory to predict data that are rather complex and need some flexibility in analysis (Ali et al., 2022). These ARIMA (p, d, q) models can be formulated by the differential equation defined in the following form: (Ibrahim, & Hayawi, 2021; Muzahem et al.,2023)

$$Y_t = C + \sum_{i=1}^{p} \varphi_i Y_i + \sum_{j=1}^{q} \theta_j a_j + \varepsilon_t \tag{1}$$

Where: $Y_t$ : the time series variable in time t, $C$ The Model constant, $\varphi_i$ Parameters of auto-regression, $\theta_j$ Moving average parameters, $\varepsilon_t$ Random error at time t

### 2.2. Bayesian Approach

Bayesian Time Series Modelling offers a powerful and flexible framework for analyzing and forecasting data that evolves and handles complex dependencies. Unlike classical frequentist methods, Bayesian time series modelling treats parameters as random variables, updating beliefs through Bayes' theorem as new data is observed (Gelman and Shalizi, 2013). This framework is particularly powerful for forecasting, anomaly detection, and parameter estimation in non-stationary (West and Harrison, 1997).

The essence of the Bayesian approach lies in Bayes' Theorem, which mathematically dictates the transformation of prior beliefs into posterior beliefs through the integration of data:

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)} \tag{2}$$

where:

- $P(\theta|y)$: **Posterior distribution** of parameters $\theta$ given data $y$,
- $P(y|\theta)$: **Likelihood function** of the time series model,
- $P(\theta)$: **Prior distribution** encoding domain knowledge,
- $P(y)$: **Marginal likelihood** (normalizing constant).

Bayesian time series models can provide a versatile toolkit for model complexities, including non-stationarity, structural changes, and latent variables. One of the widely adopted models in time series models is Bayesian ARIMA, which stands for Autoregressive Integrated Moving Average. This approach is the extension of the conventional ARIMA methodology by integrating prior probability distributions over the model parameters, allowing for a fully probabilistic analysis.

### 2.3. Wavelet Analysis (Daubechies and Symlets)

Wavelet analysis is used to extract hidden information within the data by analyzing it in the time and frequency domains. One of the most prominent types of wavelets that have a strong impact in the field of time series is Daubechies and Symlets wavelets, as they are one of the most widely used wavelets due to their ability to provide an accurate and effective analysis of unstable and time-varying signals, which makes them suitable in many applications used in the field of time series data, including monthly inflation (Ali and Jwana, 2022).

Daubechies wavelets, developed by the scientist Ingrid Daubechies, are named after her and are characterized as orthogonal wavelets that provide a high degree of smoothing and are widely used in data compression, signal analysis, and time series forecasting. It is also characterized by being based on short-wave functions, but it provides an accurate representation of the signal. This type of wave is used in the analysis and construction of time series models, especially in the field of Economics, to detect hidden patterns in the data and remove noise from them (Elias and Ali, 2025). The Symlets waveform is a modified version of the Daubechies waveform, as it was developed and some modifications were made to it to achieve a balance between symmetry and continuity, making it more suitable for some applications that require smoother signals. These wavelets have a higher continuity than Daubechies wavelets, which makes them more accurate in analyzing signals with slow changes. It is also used to purify economic data from noise to obtain more accurate forecasts, especially time series and financial data (Omer et al., 2024; Ali and Dlshad, 2021).

## 3. Efficiency Criteria

The efficiency criteria of the models estimated and used in this article are as follows:

Mean Square Error (MSE) is the metric for evaluating the performance of predictive models by calculating the average squared difference between observed values and forecasted model values (Hyndman and Athanasopoulos, 2018) Although MSE is interpretable and scale-dependent, it fails to incorporate regularization against model complexity and risks overfitting (Wulff, 2017).

$$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2 \tag{3}$$

Where: $y_t$ = Actual value at time t.

- $\hat{y}_t$ = Predicted value at time t.
- n = Number of observations.

To resolve this issue, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) incorporate both goodness-of-fit and parsimony. AIC (Akaike, 1974) minimizes information loss by balancing model likelihood and parameter count, favoring simpler models unless additional parameters significantly improve fit. BIC (Schwarz, 1978) applies a stricter penalty tied to sample size, making it more conservative for large datasets (Burnham and Anderson, 2004). Empirical studies suggest AIC excels in smaller samples, while BIC is asymptotically consistent (Claeskens and Hjort, 2008). Generally, a model with lower BIC and AIC values is chosen as it indicates a better balance between model fit and complexity. The model showing the minimum BIC and AIC is typically selected as the most suitable model among other models (Sedeeq and Meran, 2022).

$$\text{AIC} = -2k\ln(\hat{L}) + 2p \tag{4}$$

$$\text{BIC} = -2\ln(\hat{L}) + p\ln(n) \tag{5}$$

- $\hat{L}$ is the maximized value of the likelihood function for the model.
- p is the number of parameters in the model.
- n is the number of data points.
- k is a constant that depends on the distribution of the errors

Together, these metrics guide modelers in selecting specifications that generalize well (Acquah, 2010). For example, in ARIMA modelling, AIC/BIC compare lag structures, while MSE validates point forecasts (Brockwell & Davis, 2016). Their joint application ensures models are both accurate and interpretable.

## 4. Proposed Method

The proposed method includes processing the data noise and outliers of Bayesian ARIMA models based on wavelet analysis as follows:

- Apply the discrete wavelet transformation using Daubechies and Symlets wavelets for orders 10 and 15 to decompose the data of Bayesian ARIMA models into their frequency components.
- Threshold the wavelet coefficients using a method like soft thresholding, with the threshold selected via Stein's Unbiased Risk Estimate (SURE).
- SURE is a method used to choose the threshold value in wavelet denoising. SURE is an unbiased estimator of the mean squared error (MSE) between the original data and the denoised signal. It helps in determining the threshold λ that minimizes the MSE. The formula is:

$$MSE(\lambda) = \frac{1}{n}\sum_{i=1}^{n}((y_i - \hat{y}_i(\lambda))^2 - \sigma^2) \tag{6}$$

Where $y_i$ is the observed noise data for Bayesian ARIMA models, $\hat{y}_i(\lambda)$ is denoising data and $\sigma^2$ is the noise variance.

- Reconstruct the data using the inverse discrete wavelet transformation. The result is a denoised version of the original data, with reduced noise and preserved important data characteristics.
- Using denoised data in estimating parameters of Bayesian ARIMA models. Where AR and MA coefficients have a normal distribution, and the variance $\sigma^2$ has a Half-Normal distribution $\sigma_0^2$
- Since the posterior distribution does not have an analytical form, we typically use Markov Chain Monte Carlo (MCMC) methods such as Metropolis-Hastings to sample from the posterior and estimate the parameters.

- The classical Bayesian approach to ARIMA models allows for the estimation of parameters as probability distributions, with uncertainty quantified through the posterior distribution. The Metropolis-Hastings MCMC algorithm is used to sample from the posterior distribution, followed by forecasting using the sampled parameter values.

## 5. Simulation Study

To illustrate the proposed method for handling data noise and outliers, Bayesian ARIMA model data were generated using the Metropolis-Hastings algorithm for parameter estimation, with p and q set to 1, 2, and 3. Sample size (100, 300, and 500). The log-prior function imposes priors on the parameters. The AR and MA parameters are assumed to have a normal prior, while the variance parameter follows a half-normal distribution. The MCMC sampling loop uses the Metropolis-Hastings algorithm to generate posterior samples for the model parameters. The algorithm proposes new parameter values based on a random walk and then calculates the acceptance ratio (using the log-posterior) to decide whether to accept the new values. The values of the assumed parameters of the models are shown in Table 1:

**Table 1. The values of the assumed parameters of the models**

| Model | AR | | | MR | | | Variance |
|---|---|---|---|---|---|---|---|
| ARIMA (1, 0, 1) | 0.5 | | | 0.3 | | | 0.5 |
| ARIMA (2, 0, 1) | 0.5 | 0.3 | | 0.3 | | | 0.5 |
| ARIMA (1, 0, 2) | 0.5 | | | 0.4 | 0.2 | | 0.5 |
| ARIMA (2, 0, 2) | 0.5 | 0.3 | | 0.4 | 0.2 | | 0.5 |
| ARIMA (3, 0, 2) | 0.5 | 0.3 | 0.2 | 0.4 | 0.2 | | 0.5 |
| ARIMA (3, 0, 3) | 0.5 | 0.3 | 0.2 | 0.4 | 0.2 | 0.1 | 0.5 |

Simulation experiments were conducted 1,000 times, and the averages of the criteria (MSE, AIC, and BIC) for both the classical Bayesian ARIMA and the proposed methods (Daubechies 10, 15, and Symlets 10, 15) are presented in Tables 2-7.

**Table 2. The results of the criteria for the ARIMA (1, 0, 1) Model**

| Method | Sample Size | MSE | AIC | BIC |
|---|---|---|---|---|
| Classical | 100 | 0.8688 | -301.8327 | -294.0172 |
| Daubechies 10 | | 0.6480 | -399.1264 | -391.3109 |
| Daubechies 15 | | 0.2013 | -393.8501 | -386.0346 |
| Symlets 10 | | 0.4284 | -395.0623 | -387.2468 |
| Symlets 15 | | 0.3034 | -379.1231 | -371.3076 |
| Classical | 300 | 0.4925 | -1084.5042 | -1073.3928 |
| Daubechies 10 | | 0.4320 | -3308.0035 | -3296.8922 |
| Daubechies 15 | | 0.3903 | -1379.0454 | -1367.9341 |
| Symlets 10 | | 0.3948 | -1419.0067 | -1407.8953 |
| Symlets 15 | | 0.3640 | -3497.6115 | -3486.5001 |
| Classical | 500 | 0.6682 | -1494.9011 | -1482.2573 |
| Daubechies 10 | | 0.4120 | -1540.1903 | -1527.5464 |
| Daubechies 15 | | 0.4581 | -1537.4481 | -1524.8043 |
| Symlets 10 | | 0.3307 | -1706.7786 | -1694.1348 |
| Symlets 15 | | 0.4253 | -1811.4462 | -1798.8023 |

**Table 3. The results of the criteria for the ARIMA (2, 0, 1) Model**

| Method | Sample Size | MSE | AIC | BIC |
|---|---|---|---|---|
| Classical | 100 | 0.9168 | -2041.7150 | -2031.2944 |
| Daubechies 10 | | 0.6475 | -2132.1766 | -2121.7559 |
| Daubechies 15 | | 0.4977 | -2112.0655 | -2101.6448 |
| Symlets 10 | | 0.4955 | -2132.6631 | -2122.2424 |
| Symlets 15 | | 0.6072 | -2118.7471 | -2108.3264 |
| Classical | 300 | 0.8075 | -10202.7425 | -10187.9274 |
| Daubechies 10 | | 0.3788 | -11039.0046 | -11024.1894 |
| Daubechies 15 | | 0.3406 | -10551.3354 | -10536.5203 |
| Symlets 10 | | 0.3739 | -10292.0650 | -10277.2499 |
| Symlets 15 | | 0.3726 | -10937.4372 | -10922.6220 |
| Classical | 500 | 0.5114 | -11356.6000 | -11339.7416 |
| Daubechies 10 | | 0.4113 | -12377.3786 | -12360.5202 |
| Daubechies 15 | | 0.3523 | -12388.1471 | -12371.2886 |
| Symlets 10 | | 0.3769 | -12548.7368 | -12531.8783 |
| Symlets 15 | | 0.3719 | -12340.7826 | -12323.9242 |

**Table 4. The results of the criteria for the ARIMA (1, 0, 2) Model**

| Method | Sample Size | MSE | AIC | BIC |
|---|---|---|---|---|
| Classical | 100 | 0.7140 | -155.6016 | -145.1809 |
| Daubechies 10 | | 0.3623 | -234.0992 | -223.6785 |
| Daubechies 15 | | 0.3847 | -241.8868 | -231.4661 |
| Symlets 10 | | 0.3878 | -244.7666 | -234.3459 |
| Symlets 15 | | 0.4612 | -244.0669 | -233.6462 |
| Classical | 300 | 1.1977 | -2585.3395 | -2570.5243 |
| Daubechies 10 | | 0.8912 | -3669.7764 | -3654.9612 |
| Daubechies 15 | | 0.9049 | -3966.3979 | -3951.5828 |
| Symlets 10 | | 0.9929 | -3987.1387 | -3972.3236 |
| Symlets 15 | | 0.8276 | -3590.3261 | -3575.5109 |
| Classical | 500 | 0.7752 | -882.3624 | -865.5040 |
| Daubechies 10 | | 0.4240 | -1259.6261 | -1242.7677 |
| Daubechies 15 | | 0.2412 | -1185.4212 | -1168.5628 |

| | | | | |
|---|---|---|---|---|
| Symlets 10 | | 0.2462 | -1167.3151 | -1150.4567 |
| Symlets 15 | | 0.2079 | -1283.8547 | -1266.9962 |

**Table 5. The results of the criteria for the ARIMA (2, 0, 2) Model**

| Method | Sample Size | MSE | AIC | BIC |
|---|---|---|---|---|
| Classical | 100 | 0.6131 | -1244.0193 | -1230.9934 |
| Daubechies 10 | | 0.4228 | -1929.2745 | -1916.2487 |
| Daubechies 15 | | 0.3719 | -1454.4839 | -1441.4581 |
| Symlets 10 | | 0.4613 | -2174.6927 | -2161.6669 |
| Symlets 15 | | 0.3676 | -1292.0689 | -1279.0431 |
| Classical | 300 | 1.1191 | -20119.5890 | -20101.0701 |
| Daubechies 10 | | 0.9488 | -20789.4230 | -20770.9041 |
| Daubechies 15 | | 0.9088 | -21154.5231 | -21136.0042 |
| Symlets 10 | | 0.9418 | -21141.7011 | -21123.1822 |
| Symlets 15 | | 0.9106 | -20349.7621 | -20331.2431 |
| Classical | 500 | 0.2839 | -19587.9303 | -19566.8572 |
| Daubechies 10 | | 0.2436 | -29312.1618 | -29291.0888 |
| Daubechies 15 | | 0.2456 | -31973.9770 | -31952.9039 |
| Symlets 10 | | 0.2485 | -31381.2063 | -31360.1333 |
| Symlets 15 | | 0.2484 | -31545.2856 | -31524.2126 |

**Table 6. The results of the criteria for the ARIMA (3, 0, 2) Model**

| Method | Sample Size | MSE | AIC | BIC |
|---|---|---|---|---|
| Classical | 100 | 3.5139 | -15455.3094 | -15439.6784 |
| Daubechies 10 | | 2.0614 | -16765.6416 | -16750.0105 |
| Daubechies 15 | | 2.1475 | -16797.7117 | -16782.0807 |
| Symlets 10 | | 2.1301 | -16777.0134 | -16761.3824 |
| Symlets 15 | | 2.2219 | -16696.6014 | -16680.9703 |
| Classical | 300 | 3.9076 | -76683.6305 | -76661.4078 |
| Daubechies 10 | | 2.8844 | -78752.2730 | -78730.0503 |
| Daubechies 15 | | 3.7755 | -79032.9418 | -79010.7191 |
| Symlets 10 | | 3.2006 | -78711.6962 | -78689.4735 |
| Symlets 15 | | 3.2271 | -78589.5016 | -78567.2789 |
| Classical | 500 | 2.5609 | -217500.2641 | -217474.9764 |
| Daubechies 10 | | 1.3406 | -234328.9620 | -234303.6744 |
| Daubechies 15 | | 1.2229 | -234532.9764 | -234507.6888 |
| Symlets 10 | | 1.7459 | -225824.6071 | -225799.3194 |
| Symlets 15 | | 2.3524 | -233592.6507 | -233567.3631 |

**Table 7. The results of the criteria for the ARIMA (3, 0, 3) Model**

| Method | Sample Size | MSE | AIC | BIC |
|---|---|---|---|---|
| Classical | 100 | 1.4966 | -20243.8571 | -20225.6209 |
| Daubechies 10 | | 1.2041 | -21469.2715 | -21451.0353 |
| Daubechies 15 | | 1.2385 | -21404.5678 | -21386.3316 |
| Symlets 10 | | 1.1067 | -21331.9679 | -21313.7317 |
| Symlets 15 | | 1.0233 | -21347.6333 | -21329.3971 |
| Classical | 300 | 0.5562 | -19667.6536 | -19641.7272 |
| Daubechies 10 | | 0.5398 | -28921.7521 | -28895.8257 |
| Daubechies 15 | | 0.5558 | -28377.6263 | -28351.6998 |
| Symlets 10 | | 0.5325 | -27989.9125 | -27963.9860 |
| Symlets 15 | | 0.5227 | -28426.6715 | -28400.7450 |
| Classical | 500 | 0.7763 | -32125.5211 | -32122.5709 |

| | | | | |
|---|---|---|---|---|
| Daubechies 10 | | 0.7080 | -33052.5155 | -33049.5653 |
| Daubechies 15 | | 0.7081 | -33064.9835 | -33062.0332 |
| Symlets 10 | | 0.7376 | -33067.9337 | -33064.9834 |
| Symlets 15 | | 0.7171 | -33030.1221 | -33027.1719 |

## 6. Discussion of simulation results

Figures 2-7 show that the proposed methods dealt with data noise. Daubechies (10 and 15) and Symlets (10 and 15) consistently outperform the classical Bayesian ARIMA method in terms of MSE, AIC, and BIC for most models and sample sizes. Daubechies 15 and Symlets 15 generally show the best performance in terms of MSE, achieving the lowest error values for many combinations of models and sample sizes (e.g., ARIMA (1,0,1) with sample size 100, ARIMA (3,0,2) with sample size 500). This suggests that higher-order wavelets (such as Daubechies 15 and Symlets 15) are more effective at denoising the data, improving the model's forecasting accuracy.

Symlets 10 and Daubechies 10 perform similarly, but in general, Symlets tend to slightly outperform Daubechies, particularly for larger models like ARIMA (3, 0, 3). These wavelet methods also demonstrate better performance in terms of AIC and BIC for models with more parameters.

The significant improvement in MSE, AIC, and BIC when utilizing wavelet-based methods indicates that these techniques effectively mitigate noise in the data. The Daubechies and Symlets wavelets, especially those with higher orders, excel at filtering outliers and reducing noise while maintaining strong predictive power, as evidenced by the lower MSE and more negative AIC and BIC values.

Daubechies 15 and Symlets 15 outperform the classical method by a significant margin across several models and sample sizes, especially for models with higher-order AR and MA coefficients. This indicates that wavelet-based denoising is beneficial for more complex time series with intricate dynamics.

The performance of Daubechies 10 and Symlets 10 shows improvements in MSE compared to the classical method, but they do not perform as well as their higher-order counterparts, indicating that denoising effectiveness improves with wavelet order.

ARIMA (1,0,1) and ARIMA (1,0,2), proposed methods significantly outperform classical Bayesian ARIMA, especially when the sample size is small (100). This indicates that wavelet analysis methods can help enhance the model’s fitness and reduce overfitting, especially in cases where the data is noisy or limited.

The complex models like ARIMA (3, 0, 3) and proposed wavelet methods continue to show an advantage, but the performance gap is narrower compared to simpler models (such as ARIMA (1, 0, 1) and ARIMA (2, 0, 1)). This is likely because the higher number of parameters in these models means that denoising might not always lead to substantial improvements in forecast accuracy, though wavelet methods still help reduce the error.

The proposed methods show their greatest strength in larger sample sizes (300 and 500). As the dataset grows, the ability of wavelet-based methods to capture the true underlying data becomes more pronounced, leading to greater improvements in model performance. The classical Bayesian ARIMA model tends to show higher MSE, AIC, and BIC values compared to the proposed methods as the sample size increases. This suggests that the classical method is more sensitive to noise, while the proposed methods offer superior noise reduction and modelling efficiency.

MSE tends to decrease with larger sample sizes for both classical and proposed methods, indicating that larger datasets provide more reliable and stable parameter estimates. This trend is particularly noticeable for the ARIMA models with higher order (ARIMA (2, 0, 2), ARIMA (3, 0, 3)).

AIC and BIC improve (i.e., become more negative) with larger sample sizes, suggesting that the models perform better as more data is available for estimation. This is expected, as larger sample sizes allow for more precise estimates, reducing the potential for overfitting and increasing the reliability of model selection criteria.

## 7. Real Data

The time series data represents the monthly inflation in the Kurdistan Region of Iraq for the period (2009-2024), taken from the Kurdistan Region Statistics Office and represents 192 months as shown in Figure 1:

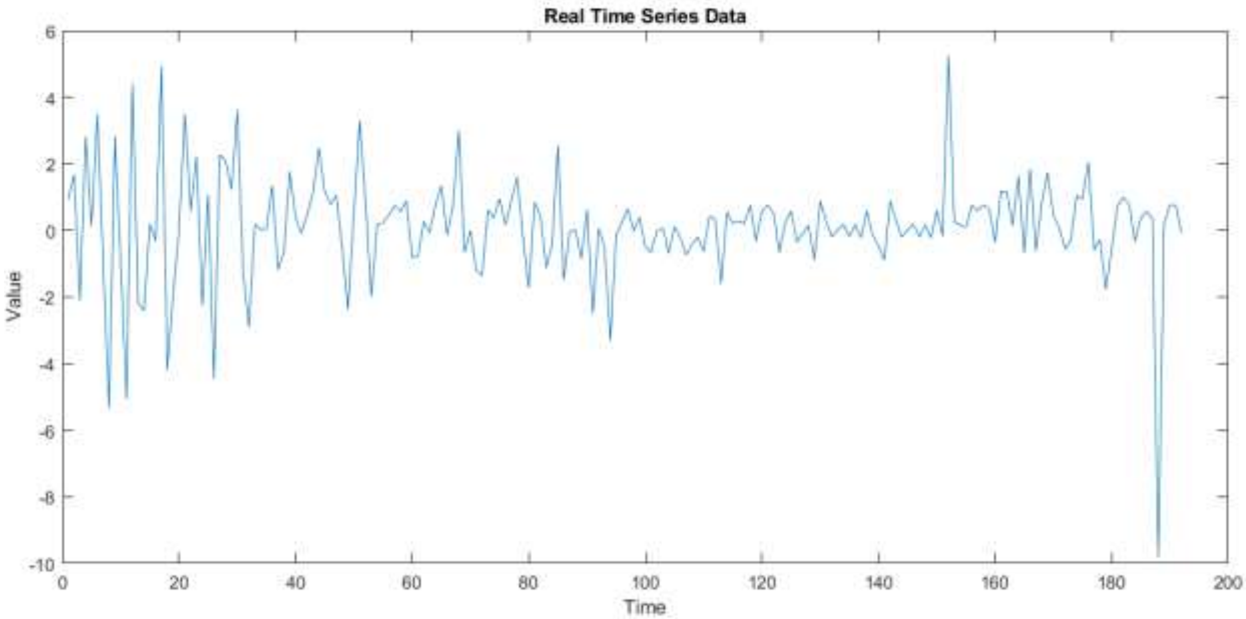


**Figure 1. Monthly Inflation in the Kurdistan Region**

The stationarity of the time series of the original and transformed data was tested using the proposed method (Symlets 15) based on the Augmented Dickey-Fuller (ADF) test, as shown in Table 8:

**Table 8. ADF Test for Monthly Inflation in the Kurdistan Region**

| Method | ADF Test Statistic | Critical Values | p-value | Decision |
|---|---|---|---|---|
| Classical | -15.5638 | -1.9423 | 0.001 | The series is stationary |
| Symlets 15 | -15.1006 | -1.9423 | 0.001 | The series is stationary |

The Monthly Inflation in the Kurdistan Region time series is stationary based on both the Classical and Symlets 15 methods of the ADF test, as the p-values are very small (0.001), and the test statistics (-15.5638 and -15.1006) are much smaller (more negative) than the critical value (-1.9423). Therefore, it is concluded that the series does not exhibit a unit root and that its properties (such as mean and variance) are constant over time.

Bayesian ARIMA models of the classical and proposed method (Symlets 15) with different orders (p and q = 1, 2, 3) were estimated and the ARIMA (1, 0, 1) model was chosen based on the least significant AIC and BIC for two methods and the significance of the estimated coefficients of those models as in Table 9.

**Table 9. Results of Time Series Analysis**

| Method | AR | MR | Variance | MSE | AIC | BIC |
|---|---|---|---|---|---|---|
| Classical | 0.440<br>(p = 0.043) | -0.130<br>(p = 0.029) | 0.067<br>(p = 0.560) | 0.2382 | -4435.345 | -4425.573 |
| Symlets 15 | 0.491<br>(p = 0.020) | 0.308<br>(p = 0.021) | 0.074<br>(p = 0.018) | 0.2145 | -4539.277 | -4529.505 |

Classical AR coefficient (0.440) with p-value = 0.043 and Symlets 15 AR coefficient (0.491) with p-value = 0.020. Both AR coefficients are statistically significant, as their p-values (0.043 and 0.020) are less than the significance level of 0.05. This means both models have strong autoregressive effects. Classical MR coefficient (-0.130) with p-value = 0.029 and Symlets 15 MR coefficient (0.308) with p-value = 0.021. Both MR coefficients are statistically significant, as their p-values (0.029 and 0.021) are also less than 0.05, indicating that the moving average effect is meaningful for both methods.

Classical variance (0.067) with p-value = 0.560 and Symlets 15 variance (0.074) with p-value = 0.018. The Symlets 15 model has a statistically significant variance (p-value = 0.018), suggesting that the variability in the residuals is explained significantly better by the Symlets 15 model than the Classical model. The Classical model has a higher p-value (0.560), meaning the variance of its residuals isn't statistically significant, implying that the residuals might not be well-explained by the model.

Classical MSE (0.2382) and Symlets 15 MSE (0.2145). The Symlets 15 model has a lower MSE (0.2145), indicating that it makes better predictions with less error compared to the Classical model (MSE = 0.2382). Classical AIC (-4435.345) and Symlets 15 AIC (-4539.277). The Symlets 15 model has a significantly lower AIC, indicating a better fit relative to its complexity compared to the Classical model. Classical BIC (-4425.573) and Symlets 15 BIC (-4529.505). Again, the Symlets 15 model has a lower BIC, indicating that it is a more optimal model compared to the Classical model.

The Classical method is still a valid model but does not perform as well as the Symlets 15 method in terms of predictive accuracy and model efficiency. Therefore, Symlets 15 would be the preferred model for forecasting the time series data.

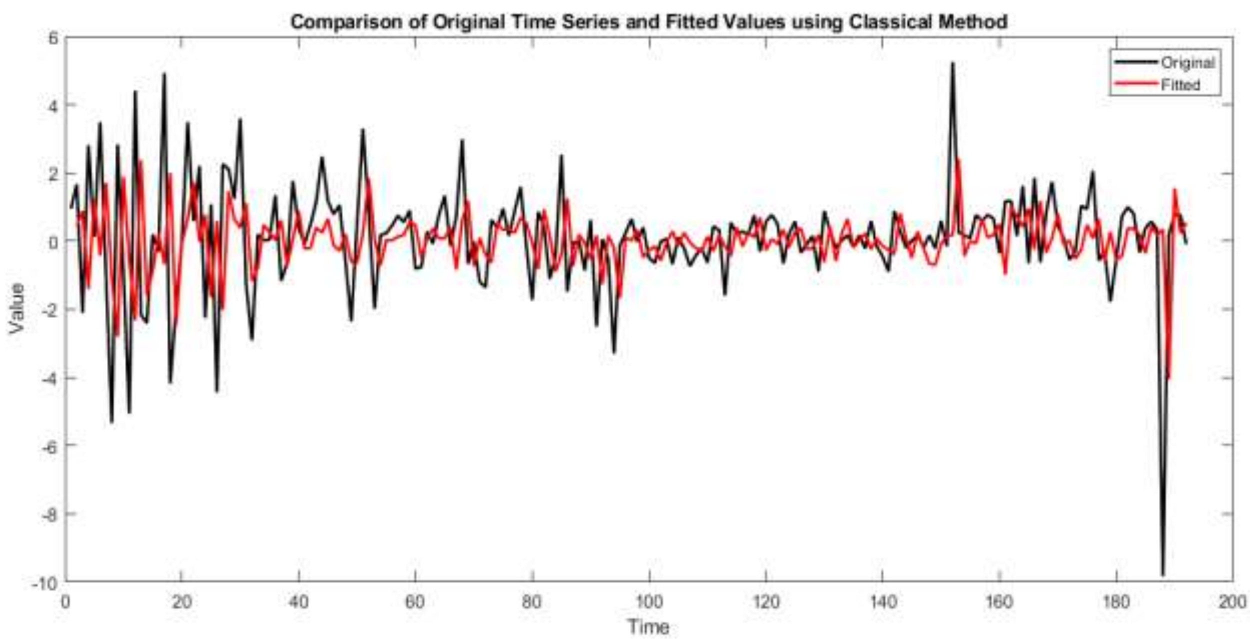


**Figure 2. Comparison of Original Time Series and Fitted Values Using Classical Method**

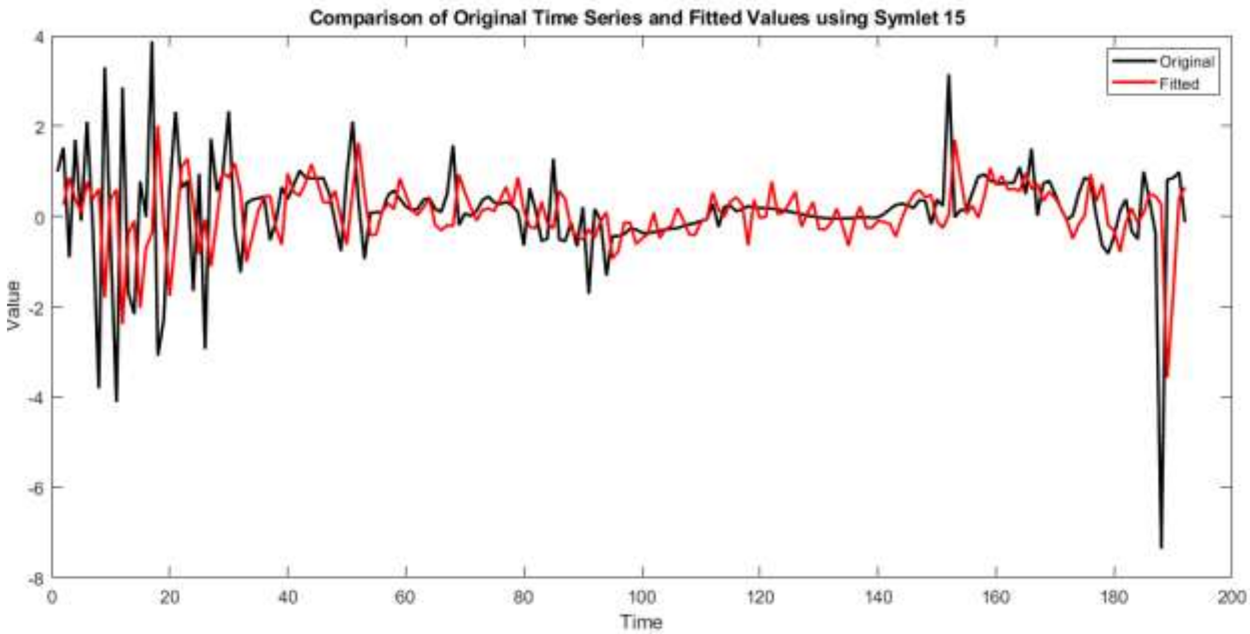


**Figure 3. Comparison of Original Time Series and Fitted Values Using Proposed Method**

Figures 2 and 3 illustrate the classical and proposed (Symlets 15) methods for the original time series data and the estimates derived from these models. The red forecasted mean line aligns with the black original data, demonstrating that the forecasted values closely match the actual data. The plot indicates that both the Bayesian ARIMA model and the proposed model effectively capture the trends and patterns in historical data, with the forecasted mean following the general direction of the series.

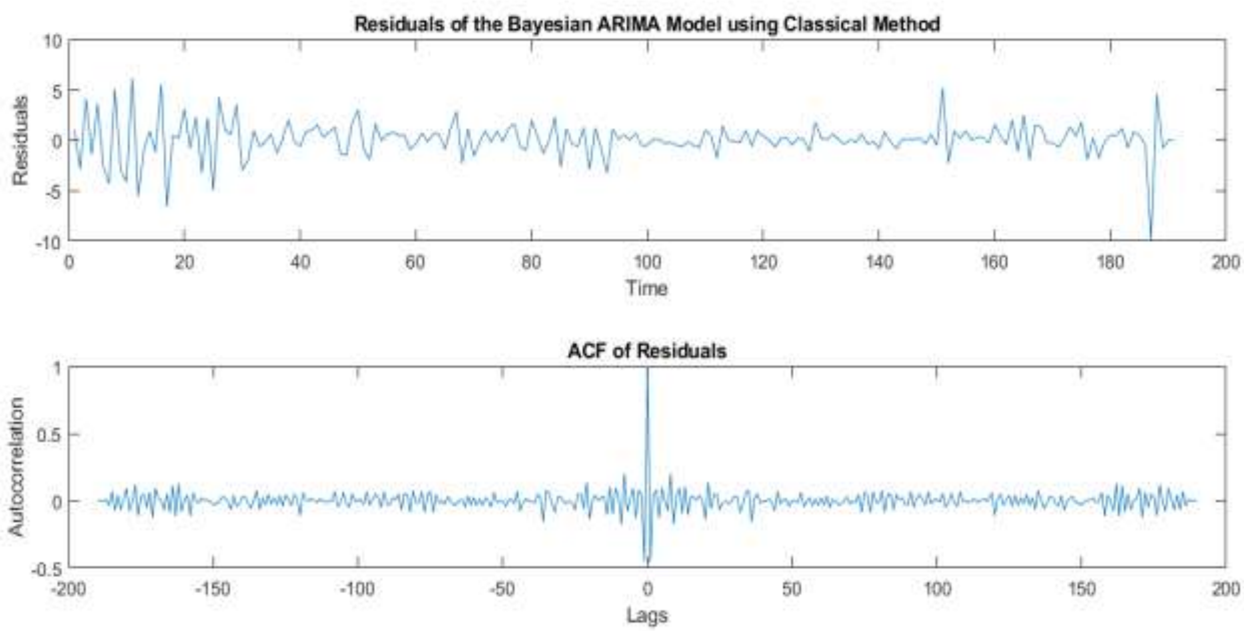


**Figure 4. Residuals and ACF of Classical Model**

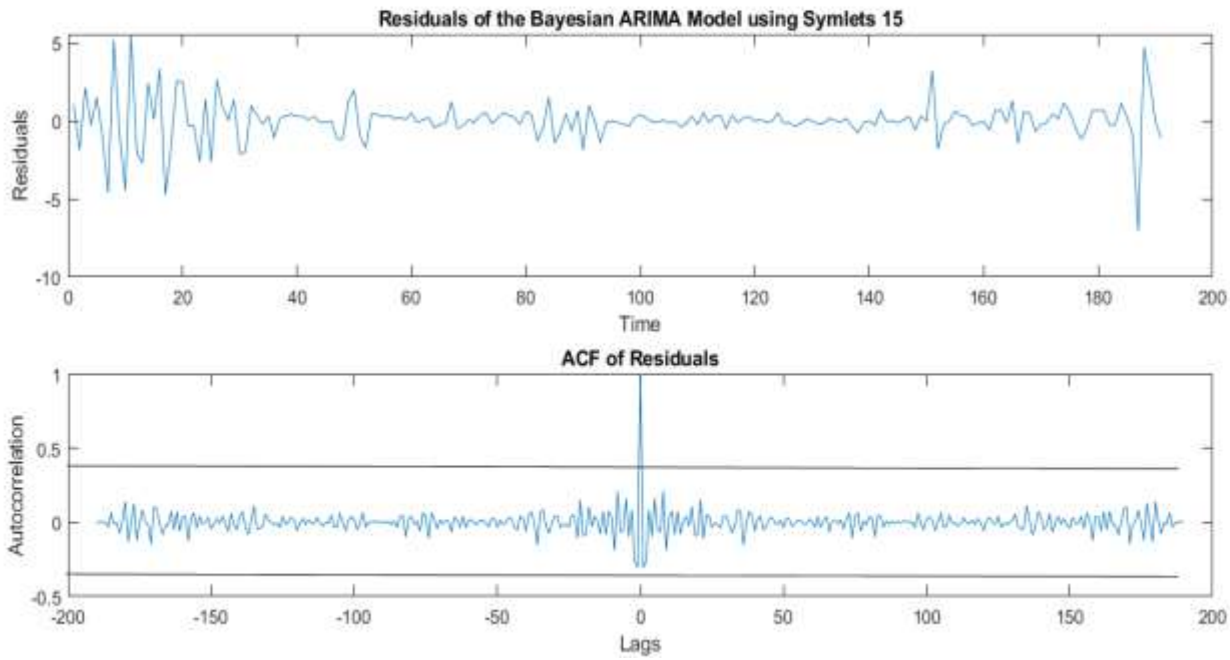


**Figure 5. Residuals and ACF of Proposed Model**

Figures 4 and 5 show that the residual plot is like random noise (white noise). This means there is no discernible pattern or trend over time. If you observe a trend or cyclical pattern in the residuals, this could indicate that the model hasn't fully captured the underlying structure of the data. Residuals are uncorrelated with each other, meaning there are no significant patterns or peaks in the residual plot. ACF plot of the residuals. If the residuals are uncorrelated, all autocorrelation values should be near zero. The results show that all autocorrelation coefficients fall within the confidence bands (blue lines), meaning that no significant autocorrelation exists among the residuals.

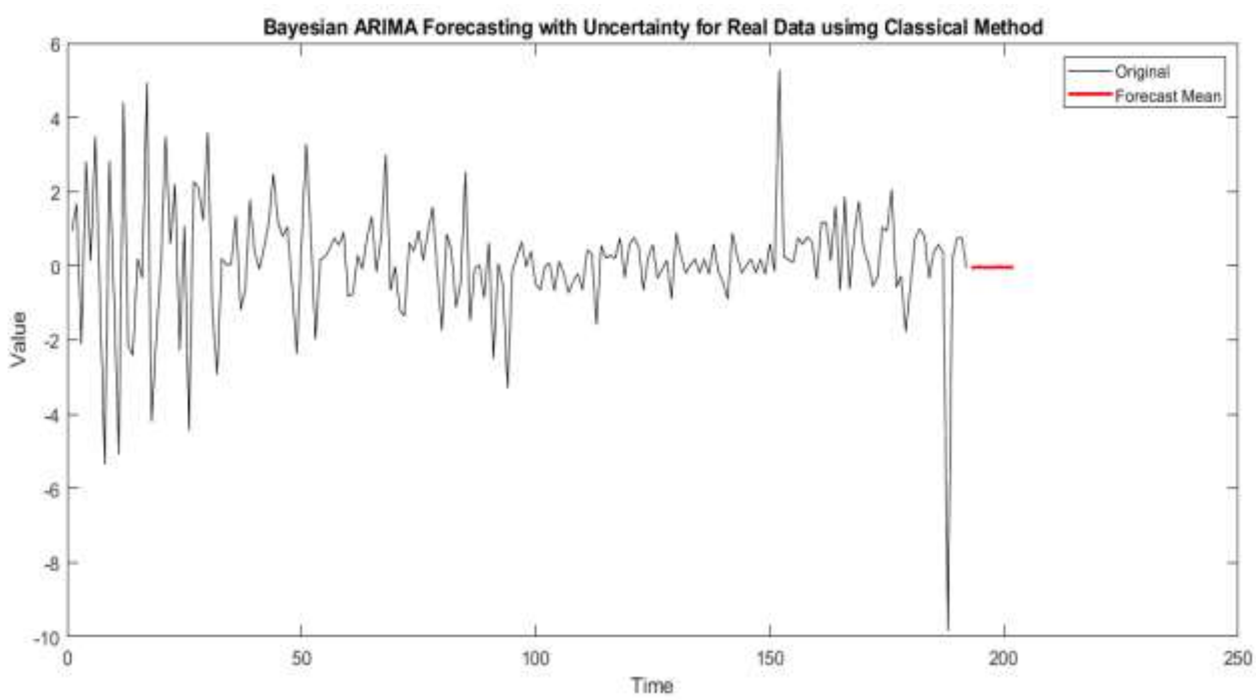


**Figure 6. Forecasting Monthly Inflation Using Classical Method**

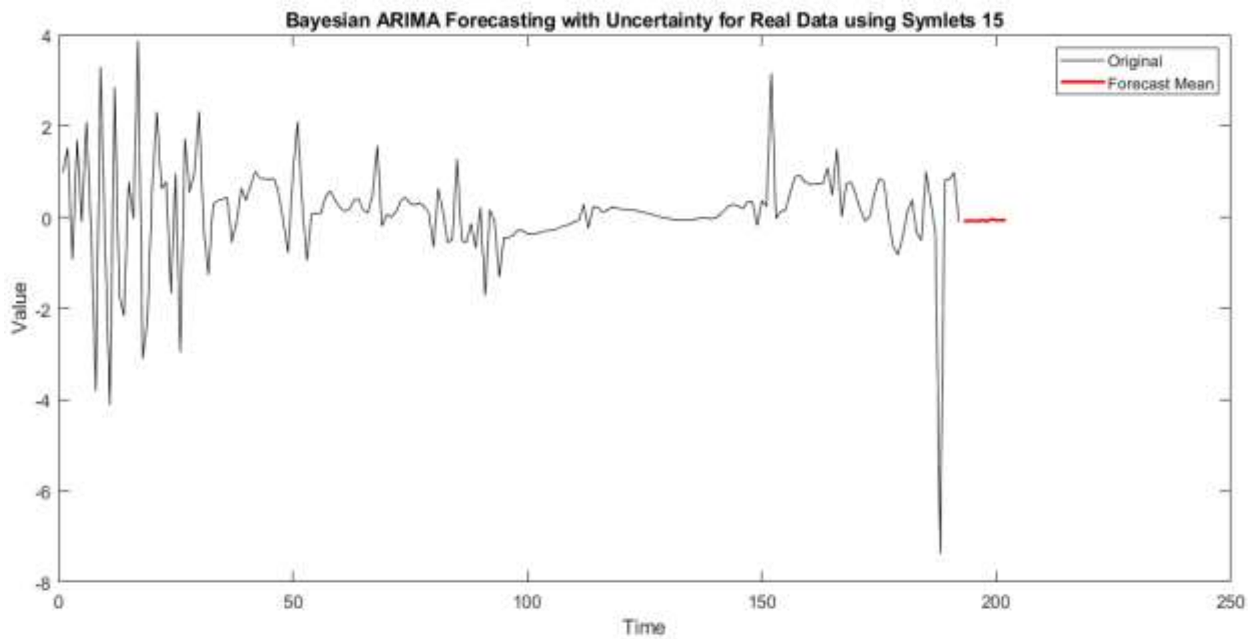


**Figure 7. Forecasting Monthly Inflation Using Proposed (Symlets 15) Method**

Figures 6 and 7 display the original series in black and the forecasted values (the predicted future values) in red, indicating a decrease in monthly inflation in the Kurdistan Region based on two methods.

**Table 10. Forecasting Monthly Inflation in the Kurdistan Region**

| Method | Classical | Symlets 15 |
|---|---|---|
| 1 | -0.0480 | -0.0809 |
| 2 | -0.0387 | -0.0807 |
| 3 | -0.0369 | -0.0760 |
| 4 | -0.0450 | -0.0842 |
| 5 | -0.0438 | -0.0613 |
| 6 | -0.0423 | -0.0880 |
| 7 | -0.0348 | -0.0396 |
| 8 | -0.0424 | -0.0653 |
| 9 | -0.0412 | -0.0619 |
| 10 | -0.0362 | -0.0540 |

Table 10 shows the forecasted monthly inflation values for the Kurdistan Region, comparing two methods: Classical ARIMA and Symlets 15 (a type of wavelet transformation method used for forecasting). The table lists the forecasted inflation values for each of the next 10 months (labelled 1 to 10). The classical method (likely a standard ARIMA model) shows forecasted inflation values that are relatively stable, hovering around -0.04 to -0.05 for most months. The values are negative, which could imply that inflation is expected to decrease slightly over the forecast period. The Symlets 15 method, which incorporates wavelet transformations, produces forecasts that generally fluctuate a bit more than the classical method. The values are also negative but appear to vary more, ranging from -0.08 to -0.04 across the forecast horizon.

## 8. Conclusion

1.In this research, the effectiveness of the proposed methods was demonstrated, particularly the use of high-order wavelets that reduce noise and improve the performance of Bayesian models. These methods help lower the average error measures and information criteria, facilitating the selection of the best model by mitigating noise and extreme values. This makes them well-suited for time series forecasting through simulation.

1. When the sample size is small (e.g., $n = 100$), reducing the influence of wavelets proves to be beneficial. However, when the sample size is large, wavelet-based methods perform significantly better than the traditional approach, namely, the Bayes method.
2. Higher-order wavelets, such as the Daubechies and Symlets wavelets at rank 15, yield better results in terms of both accuracy and forecasting, as shown by the model selection criteria. This indicates that the more complex wavelet-based method provides superior results compared to traditional methods.
3. The Bayesian and ARIMA method proposed in this study, which are based on wavelets, presents a robust framework for handling time series data. It offers considerable improvements over traditional techniques, particularly in practical applications involving outliers and noise, making it the most effective approach for time series prediction in such cases.
4. Based on the significance of coefficients (AR, MR, and variance) and the model fit metrics (MSE, AIC, and BIC) for monthly inflation in the Kurdistan Region time series, the Symlets 15 method is the superior model for this time series analysis. It provides a better fit with statistically significant results, lower MSE, and more favourable AIC and BIC values.
5. There is a decrease in monthly inflation in the Kurdistan Region.

**References**


1. Acquah, H. D. G. (2010). Comparison of Akaike information criterion (AIC) and Bayesian information criterion (BIC) in the selection of an asymmetric price relationship. J Dev Agric Econ. 2010; 2(1):1-6.
2. AKAIKE, H. 1974. A new look at the statistical model identification. IEEE transactions on automatic control, 19, 716-723.
3. Ali, Taha Hussein and Jwana Rostam Qadir. "Using Wavelet Shrinkage in the Cox Proportional Hazards Regression model (simulation study)", Iraqi Journal of Statistical Sciences, 19, 1, 2022, 17-29.
4. Ali, Taha Hussein, and Dlshad Mahmood Saleh. "COMPARISON BETWEEN WAVELET BAYESIAN AND BAYESIAN ESTIMATORS TO REMEDY CONTAMINATION IN LINEAR REGRESSION MODEL" PalArch's Journal of Archaeology of Egypt/Egyptology 18.10 (2021): 3388-3409.
5. Ali, Taha Hussein, Saman Hussein Mahmood, and Awat Sirdar Wahdi. "Using Proposed Hybrid method for neural networks and wavelet to estimate time series model." Tikrit Journal of Administration and Economics Sciences 18.57 part 3 (2022).
6. Box, G., G. Jenkins, G. Reinsel and G. Ljung (2016). Time Series Analysis Forecasting and Control. John wiley & Sons,Inc. Hoboken, New Jersey.
7. BURNHAM, K. P. & ANDERSON, D. R. 2004. Multimodel inference: understanding AIC and BIC in model selection. Sociological methods & research, 33, 261-304.
8. CLAESKENS, G. & HJORT, N. L. 2008. Model selection and model averaging. Cambridge books.
9. Elias, Intisar and Hussein Ali, T. (2025) "Choosing an Appropriate Wavelet for VARX Time Series Model Analysis", Journal of Economics and Administrative Sciences, 31(146), pp. 174–196. doi:10.33095/px3b7908.
10. GELMAN, A. & SHALIZI, C. R. 2013. Philosophy and the practice of Bayesian statistics. British Journal of Mathematical and Statistical Psychology, 66, 8-38.
11. Heyam A. Hayawi, Najlaa Saad Ibrahim Alsharabi,(2022),"Analysis of multivariate time series for some linear models by using multi-dimensional wavelet shrinkage",Periodicals of Engineering and Natural Sciences 10 (4), 120-129.
https://xjournals.com/collections/articles/Article?qt=7PzIpVggCvea+kXIRpdERyKclJE59lgkRqB5N4lmpYCnJ33rwBcEfxmhUhdkNsmyomnZWqDC91eL5xMLv7CSBA==

12. Heyam A. Hayawi ,Najlaa Saad Ibrahim , Omer Salem Ibrahim,(2021),"FORECASTING THE FUZZY HYBRID ARIMA-GARCH MODEL OF STOCK PRICES IN IRAQISTOCK EXCHANGE",Int.J.Agricult. Stat.Sci. 17 (1), 2229-2238.
13. Heyam Hayawi, Muzahem Al-Hashimi, Mohammed Alawjar.(2025)," Machine learning methods for modelling and predicting dust storms in Iraq", Statistics, Optimization And Information Computing, Vol.13, pp 1063–1075. https://connectjournals.com/03899.2021.17.2229
14. HYNDMAN, R. J. & ATHANASOPOULOS, G. 2018. Forecasting: principles and practice, O Texts.
15. Muzahem M. Al-Hashimi, Heyam A. Hayawi, Mowafaq Al-Kassab,(2023)"A Comparative Study of Traditional Methods and Hybridization for Predicting Non-Stationary Sunspot Time Series", International Journal of Mathematics and Computer Science 19 (1), 195–203.
16. Muzahem M. Al-Hashimi, Heyam A. Hayawi,(2023)," Nonlinear Model for Precipitation Forecasting in Northern Iraq using Machine Learning Algorithms", International Journal of Mathematics and Computer Science 19 (1),171-179.
17. Najlaa Saad Ibrahim, Heyam A. Hayawi,(2021),"Employment the State Space and Kalman Filter Using ARMA models",International Journal on Advanced Science Engineering Information Technology, Vol.. 11, (1),145-149. DOI: 10.18517/ijaseit.11.1.14094
18. Omer, A. W., Sedeeq, B. S., & Ali, T. H. (2024). A proposed hybrid method for Multivariate Linear Regression Model and Multivariate Wavelets (Simulation study). Polytechnic Journal of Humanities and Social Sciences, 5(1), 112-124. https://doi.org/10.25156/ptjhss.v5n 1y2024.pp112-124
19. SCHWARZ, G. 1978. Estimating the dimension of a model. The annals of statistics, 461-464.
20. SEDEEQ, B. S. & MERAN, B. W. 2022. The Use of Tobit and Logistic Regression Models to Study Factors that Affect Blood Pressure in Cardiac Patients. Cihan University-Erbil Scientific Journal, 6, 133-140.
21. WEST, M. & HARRISON, J. 1997. The dynamic linear model. Bayesian forecasting and dynamic models, 97-142.
22. WULFF, S. S. 2017. Time series analysis: Forecasting and control. Taylor & Francis.

## النمذجة البايزية للسلاسل الزمنية مع تحليل المويجات للتكهن بالتضخم الشهري

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**الخلاصة:** تتناول هذه الدراسة معالجة مشكلتي الضوضاء والقيم الشاذة في نماذج ARIMA البايزية من خلال تحليل المويجات. حيث يتم تطبيق تحويل المويجات المنفصل باستخدام مويجات (Daubechies) و (Symlets) من الرتبتين 10 و15 لتحليل بيانات نماذج ARIMA البايزية إلى مكوناتها الترددية. ثم يتم تطبيق قطع العتبة لمعاملات المويجات باستخدام طريقة مثل العتبة الناعمة(soft thresholding) ، مع اختيار قيمة العتبة باستخدام تقدير ستاين غير المتحيز للمخاطرة (Stein's unbiased risk estimate) والقاعدة الناعمة.تم إجراء تجارب محاكاة باستخدام بيانات حقيقية تمثل التضخم الشهري في إقليم كردستان العراق للفترة (2009–2024)، مع التكهن بالقيم للعشرة أشهر القادمة. يقدم نموذج ARIMA البايزي القائم على المويجات إطاراً قوياً لمعالجة بيانات السلاسل الزمنية المليئة بالضوضاء، كما يوفر تحسينات كبيرة مقارنة بالطرق الكلاسيكية، مما يجعله خياراً جذاباً للتطبيقات العملية في التنبؤ بالسلاسل الزمنية، خاصة عند التعامل مع القيم الشاذة والضوضاء.

**الكلمات المفتاحية:** نماذج ARIMA البايزية، تحليل المويجات، معالجة الضوضاء، القيم الشاذة، التنبؤ بالسلاسل الزمنية، التضخم الشهري.