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Retransformed Forecasts Back In Time Series

Abstract

The power and activity of statistical models is often improved after doing a suitable transformation, the efficacy is reflected also on analysis and forecasting using time series models. The mathematical modification of the data raises problems for the interpretation of models results and usefulness of forecasts based on transformed data, so in this paper we deal with methods for retransformed forecasts back into original scale. The application side of this paper will be in daily average data of electric power consumption in Nineveh governorate.

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[172]

1. _____ :

(2005) .

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(Liu 2006)

Retransformation

(Guerrero 1993)

Power

.Transformation

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() λ

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2. _____ :

Box and Jenkins) Univariate
ARIMA (1976

(Box and Cox 1964)

Liu and 2002) (Pankratz 1991)

: (Liu 2006) (Hudak

$$z_t = \begin{cases} \frac{(y_t^\lambda - 1)}{\lambda} & ; \text{if } \lambda \neq 0 \\ \ln(y_t) & ; \text{if } \lambda = 0 \end{cases} \quad (1)$$

:

. : Z_t

. : y_t

. $t = 1, 2, \dots, n$. $|\lambda| \leq 1$: λ

(1) λ

λ

Box and Jenkins (1976)

λ

:

Ansley, Spivey and Wroblewski(1977)

$$z_t = \begin{cases} \frac{(y_t^\lambda - 1)}{(\lambda \dot{y}^{\lambda-1})} & ; \text{if } \lambda \neq 0 \\ \dot{y} \ln(y_t) & ; \text{if } \lambda = 0 \end{cases} \quad (2)$$

:

. : $\dot{y} = (y_1 y_2 \cdots y_n)^{\frac{1}{n}}$

...

First Type (2) (1)

:

$$z_t = \begin{cases} y_t^\lambda & ; \text{ if } \lambda \neq 0 \\ \ln y_t & ; \text{ if } \lambda = 0 \end{cases} \quad (3)$$

. (Liu 2006)

Maximum Likelihood Function

Box-Cox

 λ

. (Guerrero 1993)

Stabilize

 λ :

Improve The Efficacy Of The Model

: Accuracy Of The Forecasts

 $\lambda = 0$) λ ()

.(Liu 2006) (

:_____ .3

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. (Liu 2006)

(Gurrero 1993)

. (Liu 2006)

"Naive Forecasting" "

Forecast Errors

Cost Function

.(Granger and Newbold 1986)

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$\hat{Z}_n^{(l)}$

t=n

Z_{n+1}

Box-

$\hat{Z}_n^{(l)}$

: Cox

$$\hat{y}_n^{(l)} = T^{-1}(\hat{z}_n^{(l)}) = \begin{cases} (\lambda z_n^{(l)} + 1)^{\frac{1}{\lambda}} & ; \text{if } \lambda \neq 0 \\ e^{\hat{z}_n^{(l)}} & ; \text{if } \lambda = 0 \end{cases} \quad (4)$$

(Non Box-Cox)

$\hat{Z}_n^{(l)}$

:

$$\hat{y}_n^{(l)} = T^{-1}(\hat{z}_n^{(l)}) = \begin{cases} (\lambda (\hat{z}_n^{(l)})^{\lambda-1} + 1)^{\frac{1}{\lambda}} & ; \text{if } \lambda \neq 0 \\ e^{\hat{z}_n^{(l)}} & ; \text{if } \lambda = 0 \end{cases} \quad (5)$$

$\hat{Z}_n^{(l)}$

:

$$\hat{y}_n^{(l)} = T^{-1}(\hat{z}_n^{(l)}) = \begin{cases} \hat{z}_n^{(l)\frac{1}{\lambda}} & ; \text{if } \lambda \neq 0 \\ e^{\hat{z}_n^{(l)}} & ; \text{if } \lambda = 0 \end{cases} \quad (6)$$

:

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:

Power Transformations

$$z_t = T(y_t) \quad (7)$$

:

: Z_t

$$y_t = T^{-1}(z_t) \tag{8}$$

$$\bar{y}_n^{(l)} = T^{-1}(\hat{z}_n^{(l)}) . C_\lambda^{(l)} \tag{6-3}$$

$$\bar{y}_n^{(l)} = T^{-1}(\hat{z}_n^{(l)}) . C_\lambda^{(l)} \tag{9}$$

$$C_\lambda^{(l)} = \begin{cases} \left\{ 0.5 + 0.5 \left[\frac{1 + 2(\lambda^{-1} - 1) \sigma_j^2}{\mu_j^2} \right]^{\frac{1}{2}} \right\}^{\frac{1}{\lambda}} & ; \text{ if } \lambda \neq 0 \\ \exp \left(\frac{\sigma_j^2}{2} \right) & ; \text{ if } \lambda = 0 \end{cases} \tag{10}$$

$$\mu_j = E_t[T(z_{t+j})]$$

$$Var [T(x)] = \{T'[E(x)]\}^2 Var(x) \tag{11}$$

$$\sigma_j^2 = \{T'[E_t(z_{t+j})]\}^2 Var_t[z_{t+j} - E(z_{t+j})] \tag{12}$$

:

$$T(z_{t+j}) = T[E_t(z_{t+j})] + T'[E_t(z_{t+j})][z_{t+j} - E_t(z_{t+j})] + \frac{T''[E_t(z_{t+j})][z_{t+j} - E_t(z_{t+j})]^2}{2} \quad (13)$$

:

$$\mu_j = T[E_t(z_{t+j})] + \frac{T''[E_t(z_{t+j})] \text{Var}[z_{t+j} - E_t(z_{t+j})]}{2} \quad (14)$$

: , (14) (11)

$$\mu_j = T[E_t(z_{t+j})] + \frac{T''[E_t(z_{t+j})] \{T'[E_t(z_{t+j})]\}^{-2} \sigma_j^2}{2} \quad (15)$$

:

 $T(z_t) = \log z_t$

$$\mu_j = \log[E_t(z_{t+j})] + \frac{\log g''[E_t(z_{t+j})] \{\log g'[E_t(z_{t+j})]\}^{-2} \sigma_j^2}{2} \quad (16)$$

:

$$\log[E_t(z_{t+j})] = \mu_j + \frac{\sigma_j^2}{2} \quad (17)$$

:

exp

$$E_t(z_{t+j}) = \exp\left(\mu_j + \frac{\sigma_j^2}{2}\right) \quad (18)$$

.

 $\log(z_{t+j})$

:(15)

 $\lambda \neq 0$ $T(z_t) = z_t^\lambda$

$$\mu_j = E_t^\lambda(z_{t+j}) + \frac{E_t^{\lambda''}[E_t(z_{t+j})] \{E_t^{\lambda'}[E_t(z_{t+j})]\}^{-2} \sigma_j^2}{2} \quad (19)$$

:

 $E_t(z_{t+j})$

$$E_t(z_{t+j}) = \mu_j^{\frac{1}{\lambda}} \left\{ 0.5 \pm 0.5 \left[1 + \frac{2(\lambda^{-1} - 1)\sigma_j^2}{\mu_j^2} \right]^{\frac{1}{2}} \right\}^{\frac{1}{\lambda}} \quad (20)$$

$$E_t(z_{t+j}) = T^{-1}(\mu_j). C_\lambda^{(l)} \quad (21)$$

An Approximate Debiasing Factor

$$: \quad (3) \quad (\quad)$$

$$C_\lambda^{(j)} = \begin{cases} \left\{ 0.5 + 0.5 \left[\frac{1 + 2(\lambda^{-1} - 1)\sigma_j^2}{\mu_j^2} \right]^{\frac{1}{2}} \right\}^{\frac{1}{\lambda}} & ; \text{ if } \lambda \neq 0 \\ \exp \left(\frac{\sigma_j^2}{2} \right) & ; \text{ if } \lambda = 0 \end{cases} \quad (22)$$

: (1) Box-Cox

$$C_\lambda^{(j)} = \begin{cases} \left\{ 0.5 + 0.5 \left[1 + \frac{2(\lambda^{-1} - 1)\sigma_j^2}{(\lambda^{-1} + \mu_j)^2} \right]^{\frac{1}{2}} \right\}^{\frac{1}{\lambda}} & ; \text{ if } \lambda \neq 0 \\ \exp \left(\frac{\sigma_j^2}{2} \right) & ; \text{ if } \lambda = 0 \end{cases} \quad (23)$$

: (2) Non Box-Cox

$$C_{\lambda}^{(j)} = \begin{cases} \left\{ 0.5 + 0.5 \left[1 + \frac{2(\lambda^{-1} - 1)(\sigma_j \dot{y}^{\lambda-1})^2}{(\lambda^{-1} + \mu_j \dot{y}^{\lambda-1})^2} \right]^{\frac{1}{2}} \right\}^{\frac{1}{\lambda}} & ; \text{if } \lambda \neq 0 \\ \exp \frac{\left(\frac{\sigma_j}{\bar{y}} \right)^2}{2} & ; \text{if } \lambda = 0 \end{cases} \quad (24)$$

:

$$T^{-1} \left\{ \hat{\mu}_j \pm z_{\alpha/2} \left[\hat{V} \hat{r}_t(e_{t+j}) \right]^{\frac{1}{2}} \right\} \quad (25)$$

, (22) (23) (24) (25)

$\hat{\lambda}$

λ

,

λ

.(Taylor 1986) .

,

4. _____ :

.(2008)

.(Liu 2006)

(2008)

(Liu 2006)

...

Post

Sample

Straight Retransformed
.Unbiased Retransformed

:

.1

.2

.(2005) .

.3

:

)

: (Liu 2006, 2008

Additive Outlier (AO) .1

Innovational Outlier (IO) .2

Level Shift (LS) .3

Temporary change (TC) () .4

:_____ .5

(1) 2008/8/7 2008/6/10

Post Sample

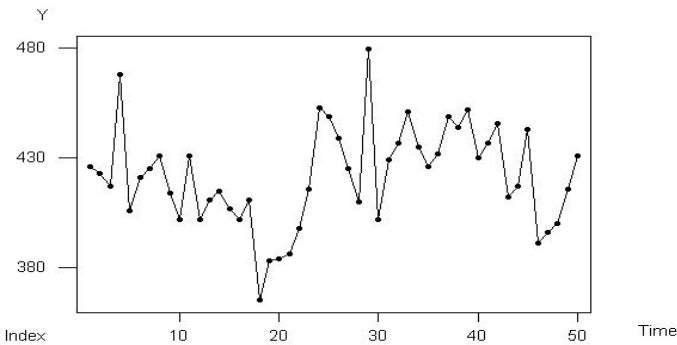
.(2) SCA

- :

Time Series Plot

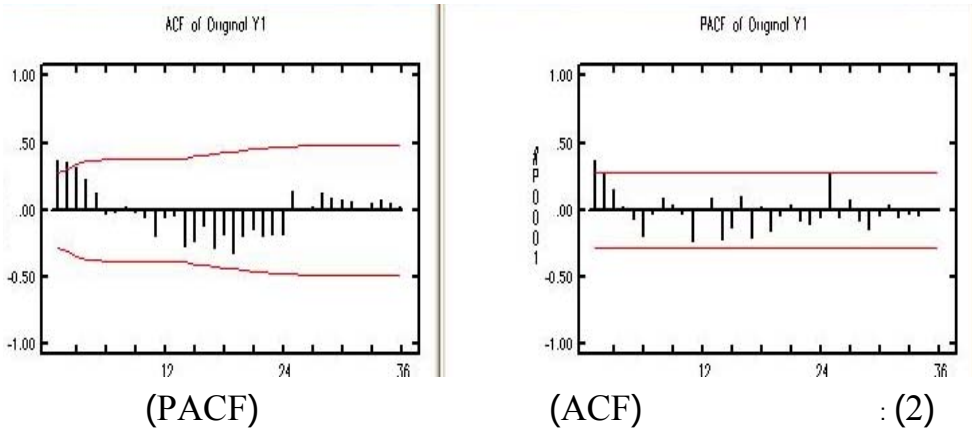
(1)

:



:(1)

:(2)



:(2)

...

UR00T Y1. METHOD DF. ORDERS 0. SPAN 1,50.
SIMPLE DICKEY-FULLER TEST
SERIES TO BE TESTED FOR UNIT ROOT(S) IS: Y1
TEST BASED ON SAMPLE SIZE: 50
PROBABILITY OF A SMALLER VALUE IS SET TO: 0.050

(<---- INTERPOLATED D-F TABLE ---->)						
CRITICAL VALUE BASED ON OLS t-STATISTIC						
ORDER	TEST	0.010	0.050	0.100	0.050	UR00T
	STATISTIC	LEVEL	LEVEL	LEVEL	LEVEL	
0	-0.18	-2.62	-1.95	-1.61	-1.95	YES

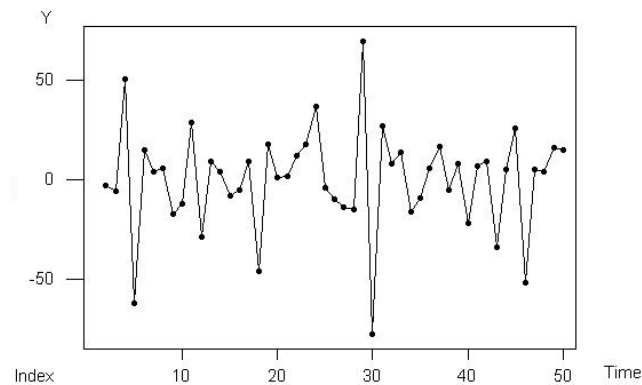
(<---- INTERPOLATED D-F TABLE ---->)						
CRITICAL VALUE BASED ON OLS AR COEFF.						
ORDER	TEST	0.010	0.050	0.100	0.050	UR00T
	STATISTIC	LEVEL	LEVEL	LEVEL	LEVEL	
0	-0.08	-12.90	-7.70	-5.50	-7.70	YES

--
DIFFERENCE Y1. NEW D1. DFORDERS 1.
DIFFERENCE ORDERS ARE (1-B)
SERIES Y1 IS DIFFERENCED, THE RESULT IS STORED IN VARIABLE D1
SERIES D1 HAS 50 ENTRIES

)

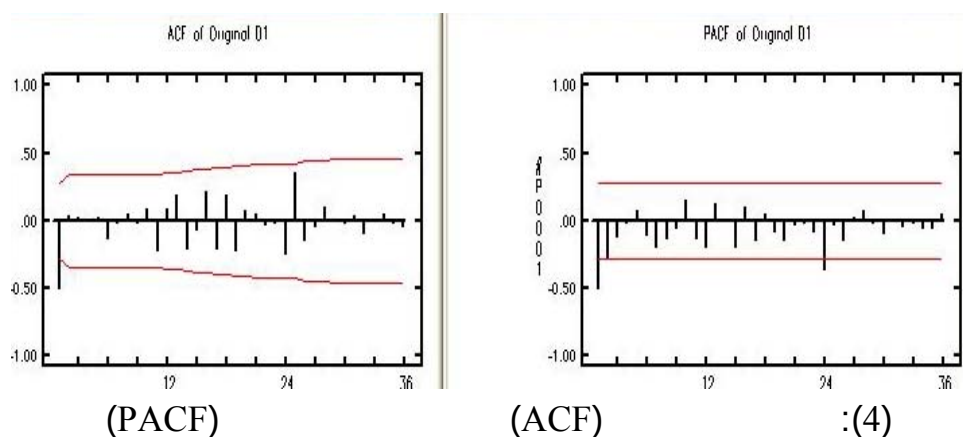
(

(3)



:(3)

:



()

URROOT D1. METHOD DF. ORDERS 0. SPAN 1,50.
SIMPLE DICKEY-FULLER TEST
SERIES TO BE TESTED FOR UNIT ROOT(S) IS: D1
TEST BASED ON SAMPLE SIZE: 49
PROBABILITY OF A SMALLER VALUE IS SET TO: 0.050

(<---- INTERPOLATED D-F TABLE ---->)

CRITICAL VALUE BASED ON OLS t-STATISTIC

ORDER	TEST	0.010	0.050	0.100	0.050	URROOT
	STATISTIC	LEVEL	LEVEL	LEVEL	LEVEL	
0	-11.81	-2.62	-1.95	-1.61	-1.95	NO

(<---- INTERPOLATED D-F TABLE ---->)

CRITICAL VALUE BASED ON OLS AR COEFF.

ORDER	TEST	0.010	0.050	0.100	0.050	URROOT
	STATISTIC	LEVEL	LEVEL	LEVEL	LEVEL	
0	-71.97	-12.86	-7.68	-5.49	-7.68	NO

--

λ

(-0.95 0.95)

λ

Root Mean Square Error (RMSE)

λ RMSE

...

[184]

RMSE

9

$$RMSE = \sqrt{\frac{1}{m} \sum_{t=1}^m (Y_t - \hat{Y}_t)^2} \quad (26)$$

:

t

: Y_t

Y_t

: $\hat{Y}_t$

: m

λ

RMSE

: (1)

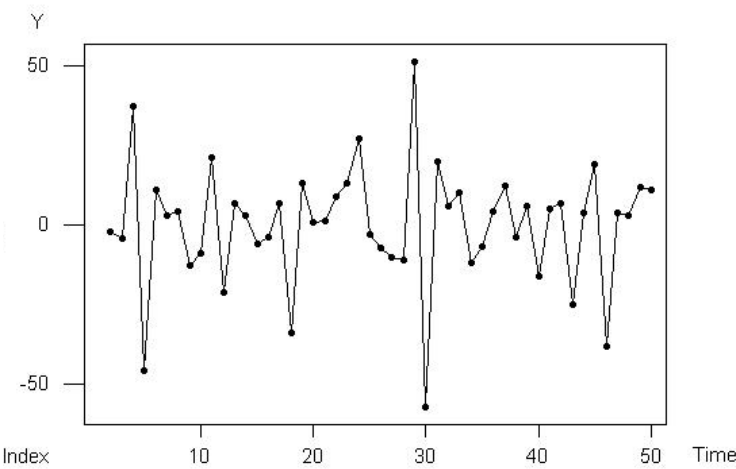
λ	RMSE	λ	RMSE
0.95	19.88885*	-0.95	21.08143
0.90	19.91616	-0.90	21.05056
0.85	19.94360	-0.85	21.01344
0.80	19.97134	-0.80	20.97798
0.75	19.99992	-0.75	20.94076
0.70	20.02722	-0.70	20.90531
0.65	20.05553	-0.65	20.87170
0.60	20.08414	-0.60	20.83640
0.55	20.11265	-0.55	20.80244
0.50	20.14167	-0.50	20.76827
0.45	20.17075	-0.45	20.73477
0.40	20.20014	-0.40	20.70022
0.35	20.22951	-0.35	20.66789
0.30	20.25929	-0.30	20.63505
0.25	20.28943	-0.25	20.60218
0.20	20.31937	-0.20	20.56990
0.15	20.34976	-0.15	20.52742
0.10	20.38058	-0.10	20.50582
0.5	20.14167	-0.5	20.76827
0.0	20.59126		

$\lambda = 0.95$

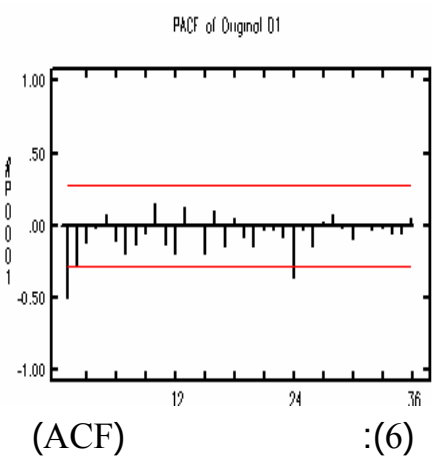
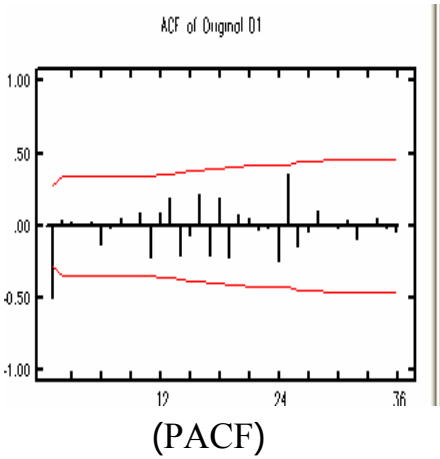
$\lambda = 0.95$ BOX-COX

$\lambda = 0.95$ BOX-COX

: (5)



:(5)



:(6)

MA(1)

(AIC_c)

: (Chatfield and Faraway 1998)

$$AIC_c = n \ln \left(\frac{S^2}{n} \right) + 2p + \frac{2(p+1)(p+2)}{n-p-2}$$

(27)

:

:n

Sum Of Square Residual

: S^2

. (Within Sample)

:P

: (2)

(AIC_c)

. AIC_c

ARMA(p,d,q)

:(2)

p	d	q	AIC_c
1	1	0	88.8911
2	1	0	87.6834
0	1	1	84.0690*
0	1	2	86.9240
1	1	1	87.1849
2	1	2	94.1811

. AIC_c

*

ARIMA(0,1,1)

(AIC_c)

.MA(1)

:

TSMODEL UTSMODEL. MODEL IS @
Y1(1) = (1)NOISE.
ESTIM UTSMODEL. HOLD RESID(_RES).
THE FOLLOWING ANALYSIS IS BASED ON TIME SPAN 1 THRU 50

NONLINEAR ESTIMATION TERMINATED DUE TO:
RELATIVE CHANGE IN (OBJECTIVE FUNCTION)**0.5 LESS THAN 0.1000D-02

SUMMARY FOR UNIVARIATE TIME SERIES MODEL - UTSMODEL

VARIABLE	TYPE OF VARIABLE	ORIGINAL OR CENTERED	DIFFERENCING					
				1				
Y1	RANDOM	ORIGINAL	(1-B	)				
PARAMETER LABEL	VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS- TRAINT	VALUE	STD ERROR	T VALUE
1	Y1	MA	1	1	NONE	.6516	.1075	6.06

EFFECTIVE NUMBER OF OBSERVATIONS . . . 49
R-SQUARE 0.117
RESIDUAL STANDARD ERROR. 0.156999E+02
--

OUTLIER UTSMODEL. TYPES A0, IO, LS. SPAN 1,50.

TIME	ESTIMATE	T-VALUE	TYPE
29	41.67	3.14	A0
24	30.67	3.06	LS
--			

() (A0) 29
9
Unbiased
Straight
Post (3)
Retransformed
Retransformed
: Sample

MA(1) : (3)

1	2	3	4	5	6
TIME	FORECAST	STD.ERROR	RETRANSFORMED UMBIASED	RETRANSFORMED STRAIGHT	POST SAMPLE
1	323.7577	15.6999	417.233	417.334	426
2	323.7577	16.6256	417.250	417.381	411
3	323.7577	17.5025	417.255	417.367	383
4	323.7577	18.3374	417.257	417.389	402
5	323.7577	19.1360	417.262	417.391	391
6	323.7577	19.9026	417.270	417.399	391
7	323.7577	20.6407	417.273	417.415	405
8	323.7577	21.3533	417.277	417.318	395
9	323.7577	22.0428	417.280	417.512	414

RMSE = 19.88873

RMSE = 19.97687

: .6

(2) .1

.2

RMSE = 19.88873

RMSE = 19.97687

.3

MA(1) .4

1. _____
" (2004) ,
2. _____
" (2008)
6 21 "(())
3. _____
".(2005)
"
4. Guerrero, V.M. (1993). "Time-series analysis supported by power transformations". Journal of forecasting 12:37-48.
5. Liu, L.-M.,2006, "Time Series Analysis and Forecasting ", 2nd ed., Scientific Computing Associates Crop., Illinois ,U.S.A..
6. Taylor, J, M, G., "The retransformed mean after a fitted power transformation " , Journal of the American Statistical Association , 81, (1986) , 114-18 .
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8. Granger, C. W. J. and Newbold, P. (1986). Forecasting Economic Time Series, 2nd ed. SanDiego: Academic Press, p. 4.
9. Ansley ,C.F., Spivey, W.A. and Wroblewski, W.J. (1977) . "A class of transformations for Box-Jenkins seasonal models". Applied Statistics 26: 173-178.
- 10.Liu, L.-M. and Hudak, G.B. , 2002," THE SCA STATISTICAL SYSTEM REFERENCE MANUAL FOR GENERAL STATISTICAL ANALYSIS" , Illinois: Scientific Computing Associates Corp. .
- 11.Pankratz ,A., 1991, "Forecasting With Dynamic Regression Models " , John Wiley and Sons , U.S.A..

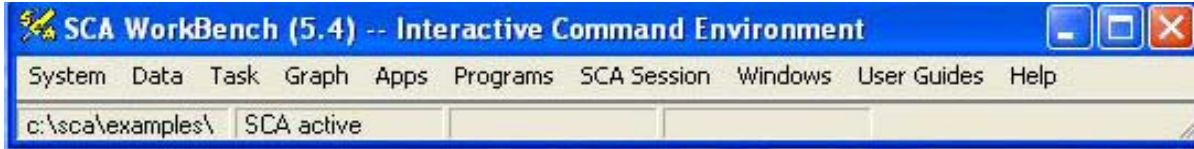
الملحق (1)

المشاهدات الحقيقية لاستهلاك الطاقة الكهربائية في مدينة الموصل للفترة 2008/6/10 ولغاية 2008/7/29:

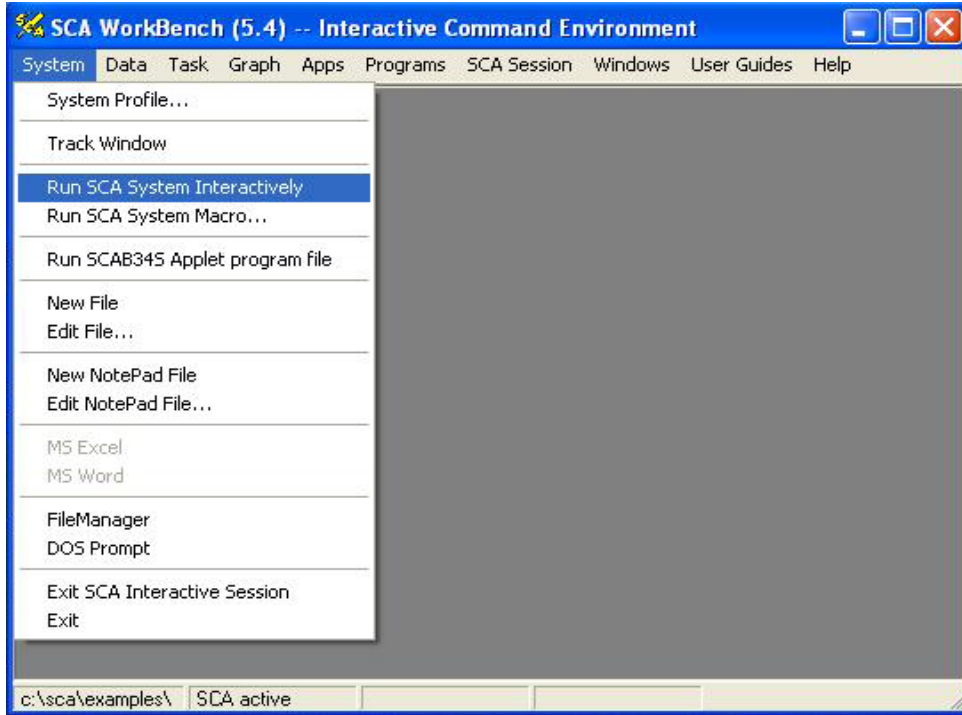
1	2008/6/10	426	26	2008/7/5	439
2	2008/6/11	423	27	2008/7/6	425
3	2008/6/12	417	28	2008/7/7	410
4	2008/6/13	468	29	2008/7/8	480
5	2008/6/14	406	30	2008/7/9	402
6	2008/6/15	421	31	2008/7/10	429
7	2008/6/16	425	32	2008/7/11	437
8	2008/6/17	431	33	2008/7/12	451
9	2008/6/18	414	34	2008/7/13	435
10	2008/6/19	402	35	2008/7/14	426
11	2008/6/20	431	36	2008/7/15	432
12	2008/6/21	402	37	2008/7/16	449
13	2008/6/22	411	38	2008/7/17	444
14	2008/6/23	415	39	2008/7/18	452
15	2008/6/24	407	40	2008/7/19	430
16	2008/6/25	402	41	2008/7/20	437
17	2008/6/26	411	42	2008/7/21	446
18	2008/6/27	365	43	2008/7/22	412
19	2008/6/28	383	44	2008/7/23	417
20	2008/6/29	384	45	2008/7/24	443
21	2008/6/30	386	46	2008/7/25	391
22	2008/7/1	398	47	2008/7/26	396
23	2008/7/2	416	48	2008/7/27	400
24	2008/7/3	453	49	2008/7/28	416
25	2008/7/4	449	50	2008/7/29	431

الملحق (2):نبذة عن نظام SCA Statistical System

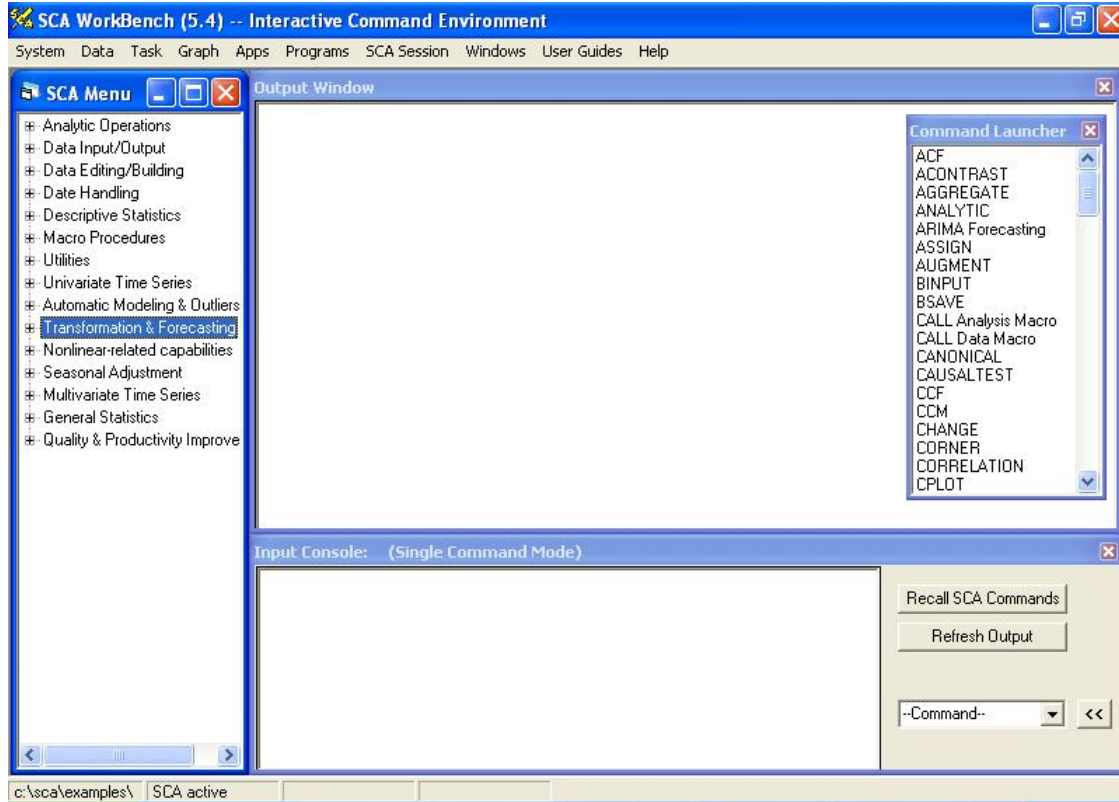
إن نظام SCA Statistical System نظام إحصائي يهتم بمجال تحليل السلاسل الزمنية خاصة فضلا عن اهتمامه بالمجالات الإحصائية الأخرى. وبتشغيل الأيقونة الخاصة بالنظام ستظهر واجهة النظام بالشكل الآتي:



وبتنفيذ الإيعاز Run SCA System Interactively من قائمة System كما في الشكل الآتي:



لتصبح واجهة النظام بالشكل الآتي:



ويمكن تنفيذ الأوامر الخاصة بالنظام بعدة أساليب منها:

1. عن طريق اختيار الإيعاز من قائمة Command Luncher والتي تحوي إيعازات النظام كافة.
2. عبر اختيار الإيعاز من قائمة SCA Menu التي تحوي كافة الإيعازات.
3. عن طريق طباعة الإيعاز بشكل صحيح بمدخلاته كافة في مربع النص الخاص بإيعازات الإدخال والمسمى Input Console.

وبعد كتابة الإيعاز بشكل صحيح يظهر تنفيذه في شاشة الإخراجات Output Windows. كما يحوي النظام على قوائم الأوامر التي من الممكن اختيارها من شريط القوائم في الجهة العليا من الواجهة.

أما بالنسبة لإدخال البيانات إلى النظام فهناك عدة طرائق منها عن طريق استيرادها من أوراق عمل خاصة بأنظمة أخرى مثل Microsoft Excel وذلك بعد تحويلها إلى أوراق عمل خاصة بنظام SCA.

أو عن طريق الإدخال المباشر في شاشة Input Console. كما يمكن إدخال البيانات إلى النظام عن طريق اختيار الإيعاز Input حيث يتم استيراد البيانات من ملفات مخزونة في الحاسوب كما في الشكل الاتي:

كما يحوي النظام على إمكانية إجراء الرسوم البيانية المختلفة وذلك عن طريق الإيعاز Graph من قائمة Descriptive Statistics وتنفيذ هذا الأمر يظهر مربع الحوار الآتي:

كما يحوي النظام على ملحقات خاصة تفيد في عمليات تحليل البيانات ومعالجتها فضلا عن شرح واف يستفاد منه في التطبيق العملي.

وللنظام عدة إصدارات منها الإصدار التعليمي Educational Edition والإصدار الاحترافي بنوعيه Professional Edition (A) and Professional Edition (B) والإصدار المتقدم Advanced Edition وإصدارات أخرى وكل إصدار له مواصفاته الخاصة وسعره الخاص ومدة انتهائه المحددة.

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