

*

:

Estimation of State Space Models by using Ridge Regression Technique with Application

Abstract

In this paper many state space models are fitted. The statistical criteria used to choose the best estimated model whose parameters estimated by the Ridge regression method. This method can be used when the predictor variables are multicollinear. The data used in this paper represented by the temperature, which is the response variable and the predictor variables effulgence, wind, sunshine and evaporation.

* مدرس/قسم الاحصاء والمعلوماتية/ كلية علوم الحاسوب والرياضيات

4/21 :

2010/3/22:

2010/

-1

.

.

-:

-1985)

/ .)

()

.

(/)

()

(2000

(²

-2

$$\begin{aligned}
 Y & \quad \underline{Y} = X \underline{\beta} + \underline{U} \\
 X & \quad (n \times 1) \\
 U & \quad ((m+1) \times 1) \quad \beta \quad (n \times m) \\
 Var(\underline{U}) &= \sigma^2 I_n \quad E(\underline{U}) = \underline{0} \quad (n \times 1)
 \end{aligned}$$

(Multicollinearity)

(2005) .

$\vdots$
 $(\quad) \quad (m) \quad \underline{X}_1, \underline{X}_2, \dots, \underline{X}_m$
 $(\quad) \quad C_1, C_2, \dots, C_m$
 (1996) :

$$\sum_{j=1}^m C_j \underline{X}_j = \underline{0} \quad \dots(1)$$

$$\begin{aligned}
 & (1) \\
 C_j \neq 0 & \quad . (1) \quad C_1 = C_2 = \dots = C_m
 \end{aligned}$$

-:

$$\underline{X}_j = \frac{1}{C_1}[C_1X_1 + C_2X_2 + ... + C_mX_m]$$

... (2)

(m)

(2)

.

:

(2005)

Exact Multicollinearity

.1

$$C_1, C_2, ..., C_m$$

(3)

$$(X'X)$$

m

$$(X'X)$$

$$|X'X| = 0$$

.

Non Exact ()

.2

Multicollinearity

:

$$a_1x_1 + a_2x_2 + a_3x_3 + ... + a_mx_m + \omega_i = 0$$

... (3)

$$i = 1, 2, ..., m$$

$$(\omega_i \neq 0)$$

$$a_1, a_2, ..., a_m$$

$$(X'X)$$

$$\begin{aligned}
 V(\hat{\beta}_{OLS}) &= S^2 (X'X)^{-1} \\
 &= \frac{adj(X'X)}{|X'X|} S^2 \quad \dots (4)
 \end{aligned}$$

(t)

(1996) :

.1

$r(x_i, x_j) = \pm 1$
 X
 unity)
 $(X'X)$ (length and scaling
 Gunst and Mason (1980)
 $r(x_i, x_j) > 0.7$
 x_j x_i

$$\begin{array}{ccc}
 x_i, x_j & & x_i, x_j \\
 & \vdots & \\
 X_i \approx -X_j & \text{or} & X_i + X_j = 0 \\
 & x_i^*, x_j^* & \\
 & & \vdots \\
 X_i \approx X_j & \text{or} & X_i - X_j = 0 \\
 & & r(x_i, x_j)
 \end{array}$$

.2

Ferrar ,Glaube (1967)
(VIF) Minquardt (1970)

$$\begin{array}{ccc}
 & \vdots & (X'X)^{-1} \\
 VIF=a_{ij}-(1-R_j^2)^{-1} & & \dots(5)
 \end{array}$$

$$\begin{array}{ccc}
 j=1,2,...,m & & \\
 R^2 & & m
 \end{array}$$

$$\begin{array}{ccc}
 & & X_j \\
 & & (5)
 \end{array}$$

$$\begin{array}{ccc}
 a_{jj} & & R_j^2
 \end{array}$$

$$X_j$$

$$\begin{array}{ccc}
 . a_{jj} = 1 & & R_j^2 = 0
 \end{array}$$

$$\begin{array}{ccc}
 x_j & & a_{jj} = VIF \geq 10
 \end{array}$$

▪

$a_{jj} \geq 4$ Gunst and Mason (1980)

X_j

(1996).

:

.1

.2

.3

.4

(أ)

(ب)

(ت)

Principle Component Analysis

-1

Ridge Regression Method

-2

-3

$(X'X)$

(kIp)

(Hoerl and Kemard(1970a)) :

$$\hat{\beta} = (X'X + kIp)^{-1} X'Y \tag{6}$$

$(p \times p)$
 $(kIp) :$

$$\frac{\beta}{m^2}$$

Kuhn – $\beta'\beta \leq m^2$

:

Tucker

$$L(\beta, C) = -(Y - X\beta)'(Y - X\beta) = K(m^2 - \beta'\beta) \tag{7}$$

...(7)

β

$L(\beta, C)$

(Davidov,2006). (6)

:

.1

$(X'X)$

$k \quad k$

$$0 < k < 1, (Ip)$$

mean square error

.

k :

.2

k

(0,1) k .3

.

(2000).

$$k$$

(2007

) .

 k

(2009) .

-4

Dynamic Control

.

(2002) .

 U_t
 X_t
 X_{t+1}
 $(t+1)$
 Y_t

State Equation

 U_t
 X_t

Observation Equation

-: SISO (Single Input Single Output)

$$X_{t+1} = AX_t + BU_t + He_t \quad \dots \quad (8a)$$

$$Y_t = CX_t + DU_t + e_t \quad \dots \quad (8b)$$

$$\text{.}(n*n) \quad :A \quad -:$$

$$\text{.}(n*n) \quad :B$$

$$\text{.}(n*1) \quad :C$$

$$:H \quad . \quad :D$$

$$\text{.}(n*1)$$

.(Nelles,2001)

-5

Loss Function .1

(Nelles, 2001).

-:

$$V = \frac{1}{2} \sum_{t=1}^N e_t^2 \quad \dots \quad (9)$$

-:

: e_t : N : V

.2

Akaik's Final Prediction Error Criterion

 FPE

1969

Akaike

-:

(Ljung,2004)

$$FPE = \frac{1 + \frac{m}{N}}{1 - \frac{m}{N}} V \quad \dots (10)$$

-:

.

: m

: FPE

Akaike's Information

.3

Criterion

(1974- 1973)

Akaike

 AIC

ARIMA

AIC

:

(Nelles, 2001) (Wei ,1990)

$$AIC = 2 p - 2 \ln L \quad \dots (11)$$

-:

.

: L

: AIC

Matlab

(Ljung, 2004)-:

$$AIC=\log\left(V\left(1+2\frac{m}{N}\right)\right)$$

... (12)

-6

(2000-1985)

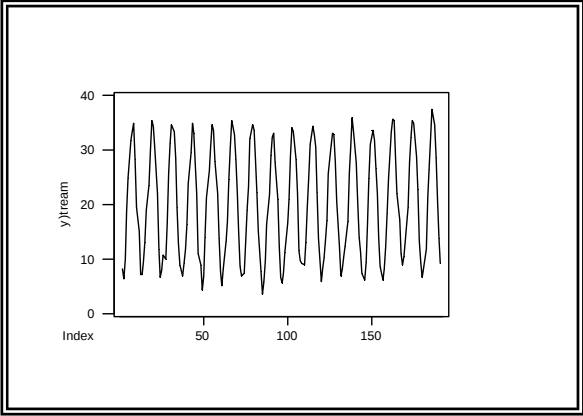
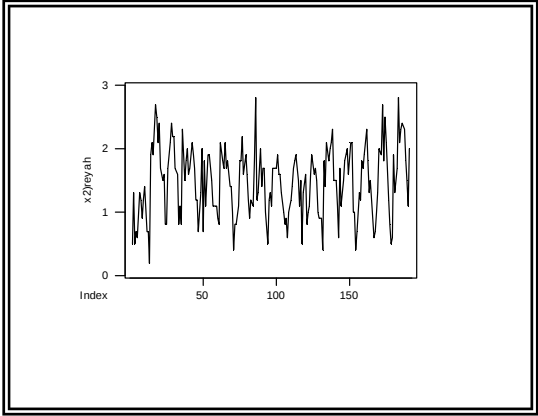
(² / .)

()

(/)

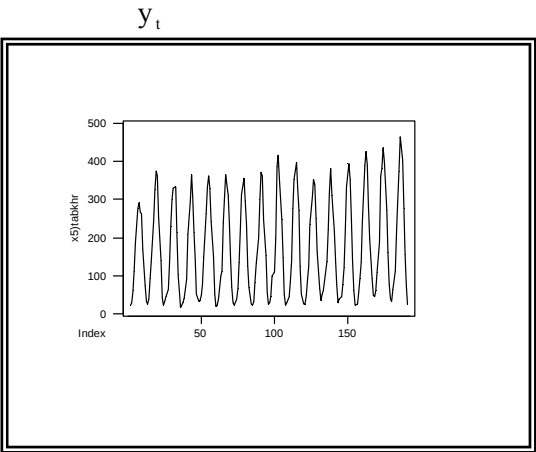
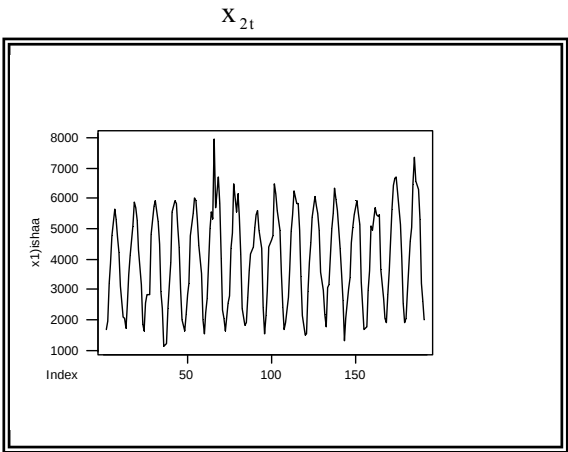
()

:



: (2)

: (1)

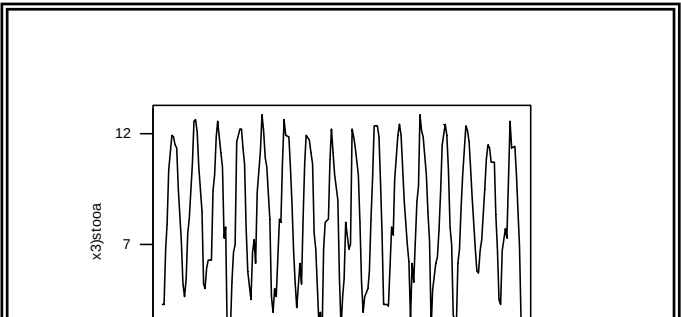


: (4)

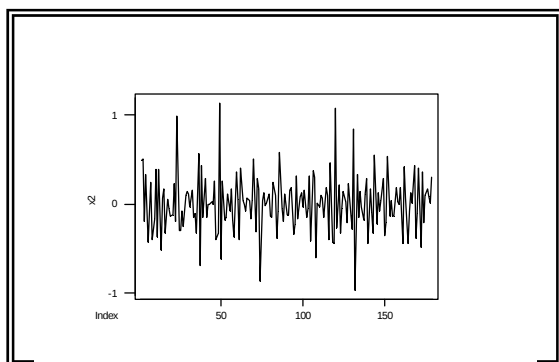
: (3)

X_{1t}

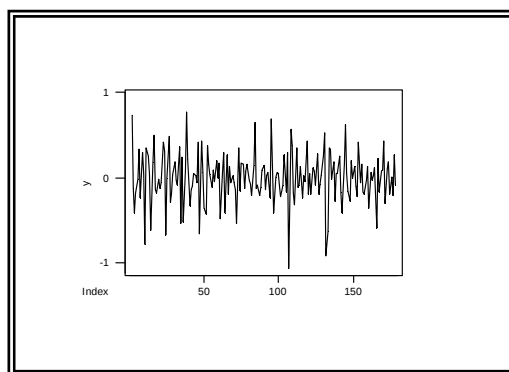
X_{4t}



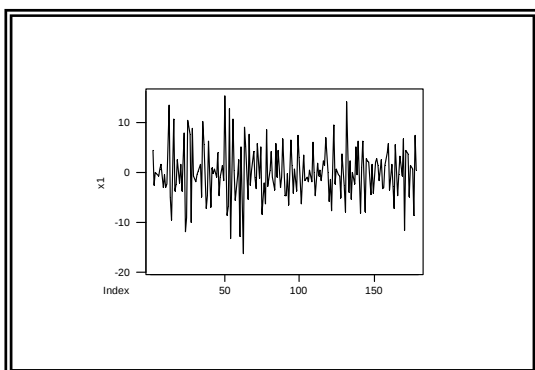
:



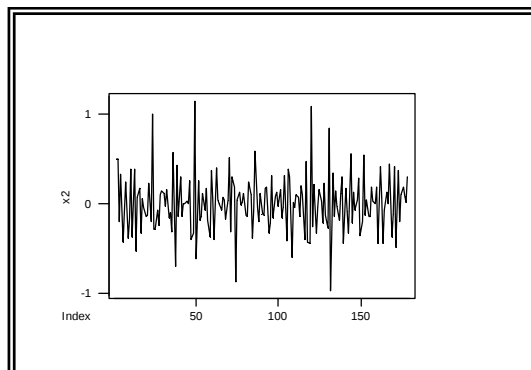
: (7)



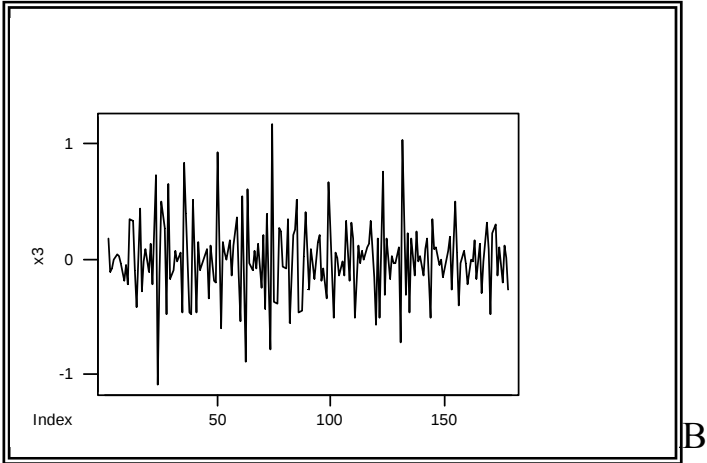
: (6)



: (9)



: (8)



: (10)

(10-1)

: (1)

	AIC	Loss fun.	FPE
1	-2.7275	0.06092	0.0653
2	-2.6674	0.06029	0.0694
3	-2.6805	0.05544	0.06858
4	-2.2995	0.07563	0.10049
5	-3.0618	0.03288	0.04697
6	-2.5689	0.05016	0.07711
7	-1.2597	0.1731	0.2867
8	-2.6260	0.04114	0.07351
9	-1.5479	0.112	0.217
10	-0.8399	0.213	0.4456

AIC

FPE

Loss Function

:

State space model :

$$A = \begin{bmatrix} -0.48633 & 0.46964 & 0.48097 & 0.25821 & 0.30226 \\ -0.62875 & -0.3991 & -0.34703 & -0.081675 & -0.082651 \\ 0.20777 & 0.10041 & 0.11191 & -0.0042911 & -1.1727 \\ 0.25077 & 0.67577 & -0.18866 & -0.41521 & -0.7686 \\ -0.056559 & 0.039169 & -0.013694 & 0.74186 & -0.26141 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0028249 & 0.079402 & 0.068167 & -0.066512 \\ -0.0037625 & 0.051898 & 0.37275 & -0.050124 \\ 0.0010686 & -0.16341 & -0.90413 & 0.12487 \\ 0.0043885 & 0.11034 & 0.2813 & -0.05409 \\ -0.0025527 & 0.052015 & 0.20093 & -0.026866 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.02384 \\ -0.39616 \\ 1.1947 \\ -0.15996 \\ -0.32353 \end{bmatrix}, \quad X(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [-1.0056 \quad 1.4557 \quad -0.15762 \quad -1.2118 \quad -0.56404]$$

D= Constant

MATLAB

ARMAX

: ARMAX

$$y_t = -1.45 y_{t-1} - 1.521 y_{t-2} - 0.6169 y_{t-3} - 0.04336 y_{t-4} + 0.3787 y_{t-5} - 0.01236 u_{1,t-1} - 0.01706 u_{1,t-2} \\ - 0.01334 u_{1,t-3} - 0.0006575 u_{1,t-4} + 0.003769 u_{1,t-5} - 0.1416 u_{2,t-1} - 0.2253 u_{2,t-2} - 0.05502 u_{2,t-3} \\ + 0.132 u_{2,t-4} + 0.1569 u_{2,t-5} + 0.1623 u_{3,t-1} + 0.2561 u_{3,t-2} + 0.5577 u_{3,t-3} + 0.5447 u_{3,t-4} + 0.4863 u_{3,t-5} \\ + 0.05494 u_{4,t-1} + 0.1212 u_{4,t-2} + 0.02225 u_{4,t-3} - 0.05795 u_{4,t-4} - 0.1092 u_{4,t-5} + a_t + 1.038 a_{t-1} \\ + 0.5173 a_{t-2} - 0.7052 a_{t-3} - 0.7709 a_{t-4} - 0.7337 a_{t-5}$$

... (13)

)

(

:

$$R = \begin{bmatrix} 1.00000 & 0.88609 & 0.34162 & 0.94002 & 0.96552 \\ 0.88609 & 1.00000 & 0.49878 & 0.92472 & 0.91152 \\ 0.34162 & 0.49878 & 1.00000 & 0.34420 & 0.45028 \\ 0.94002 & 0.92472 & 0.34420 & 1.00000 & 0.93539 \\ 0.96552 & 0.91152 & 0.45028 & 0.93539 & 1.00000 \end{bmatrix}$$

0.7

0.96552 0.94002 0.88609

:

(2)

VIF	المتغيرات
9.6	الإشعاع
1.6	الرياح
13.0	السطوع
9.7	التبخّر

VIF

VIF

Gunst and mason (1980)

4

() k

k=0.70

k

:

MINITAB

$$y = 4.38803 + 0.00019x_1 - 1.46362x_2 + 0.9910k_3 + 0.05996x_4 \quad \dots (14)$$

...

:

: (4)

Loss Fun.	FPE	AIC	MSE	
0.5891	0.6587	2516.7	56529	
0.03288	0.04697	860.3	4.959	

.

-7

:

.1

.

.2

.4

-8

-9

1. (2002):

"

2. (1996):

"

3. (2005):

"

4.

H.S.L. (2007):

29

"

86 . 179-171

5. (2000):

"

6. (2009):

"

-

.2009/12/7-6

7. Davidov, O., (2006): "**Constrained Estimation and the Theorem of KUHN-TUCKER**", Hindawi Publishing Corporation, Journal of Applied Mathematics and Decision Sciences, Article ID 92970, PP.(1-13).
8. Horel, A.E & Kennard, R.W., (1970a): "**Ridge Regression: Biased Estimation for Nonorthogonal Problem** "; Technometrics, No.1.12.O P.Cit.P.58.OP.Cit.PP.69-80.
9. Nelles, O., (2001): "**Nonlinear System Identification from Classical Approach to Neural Network and Fuzzy Models**", Springer Verlag Berlin Heidelberg Germany.
10. Ljung, L., (2004): "**System Identification Toolbox for use with MATLAB**", 6.0 Math works Inc.
11. Wei, W.W.S., (1990): "**Time Series Analysis Univariate and Multivariate Methods**", Addison Wesley Publishing Company Inc. The advanced Book Program California U.S.A.

ت	Y	X1	X2	X3	X4	ت	Y	X1	X2	X3	X4
1	8.2	1687	0.5	4.3	21.6	98	7.4	2951	1.7	5.4	44.2
2	6.4	1977	1.3	4.3	29.7	99	11.3	4392	1.7	8.0	100.8
3	10.0	3203	0.5	6.8	59.7	100	16.6	4627	1.7	6.8	109.5
4	18.6	3822	0.7	7.9	113.6	101	21.1	4754	1.9	7.0	197.3
5	24.9	4769	0.6	10.3	186.3	102	29.1	6449	1.6	12.2	386.6
6	31.8	5641	1.3	11.9	277.1	103	34.0	6138	1.6	11.8	418.2
7	33.3	5330	1.2	11.8	292.0	104	33.4	5606	1.3	11.3	353.6
8	34.9	4718	0.9	11.5	265.0	105	28.1	4943	1.0	10.1	246.5
9	28.1	4200	1.2	11.3	261.8	106	22.0	3542	0.8	8.0	147.7
10	19.4	3129	1.4	9.5	169.0	107	11.5	2411	0.9	5.3	52.5
11	15.2	2096	0.7	7.1	73.6	108	9.8	1666	0.6	3.9	24.0
12	7.3	2036	0.7	5.2	33.2	109	9.2	1899	1.0	4.6	28.6
13	7.3	1742	0.2	4.6	24.2	110	8.9	2724	1.2	5.0	46.4
14	9.6	2451	1.9	5.4	40.0	111	13.2	3666	1.4	5.8	83.3
15	13.0	3494	2.1	7.5	92.0	112	19.4	4881	1.7	8.1	137.0
16	18.9	4134	1.9	8.2	139.3	113	24.8	5446	1.8	10.0	276.7
17	23.5	5074	2.7	10.6	243.7	114	31.0	6237	1.9	12.3	352.0
18	30.1	5867	2.5	12.5	325.4	115	34.3	5836	1.5	12.3	397.0
19	35.5	5669	2.1	12.6	373.7	116	32.7	5836	1.1	11.8	329.9
20	34.4	5269	2.4	12.0	361.7	117	30.4	4932	1.5	9.7	272.8
21	30.8	4308	1.7	10.5	252.3	118	22.4	3415	0.5	7.5	124.1
22	22.1	3166	1.5	8.5	139.6	119	13.9	2129	1.3	4.3	51.2
23	11.7	1862	1.6	5.2	41.5	120	5.8	1480	1.6	4.3	25.3
24	6.6	1614	0.8	5.0	21.3	121	8.3	1553	0.8	4.2	25.2
25	8.0	2541	0.8	5.9	33.0	122	10.3	2928	1.0	6.0	46.9
26	10.7	2843	1.7	6.3	46.0	123	13.5	3907	1.1	7.8	87.2
27	10.0	2843	2.1	6.3	63.2	124	17.1	4528	1.6	7.4	124.4
28	16.7	4808	2.4	9.4	139.6	125	25.6	5344	1.9	9.9	233.9
29	25.9	5280	2.2	10.1	230.8	126	30.7	6029	1.6	11.9	307.9
30	31.3	5728	2.2	11.8	301.2	127	33.0	5713	1.7	12.4	352.8
31	34.6	5931	1.7	12.5	330.2	128	32.9	5483	1.5	11.9	339.2
32	33.3	5216	1.6	11.1	332.9	129	28.0	4926	1.0	10.4	250.4
33	28.4	4486	0.8	10.5	216.3	130	20.6	3568	0.9	8.9	159.1
34	19.6	2906	1.1	7.3	101.7	131	12.2	2967	0.9	7.0	68.9
35	12.7	2373	0.8	7.8	56.2	132	6.8	2192	0.4	6.2	34.2
36	9.1	1150	2.3	2.8	17.0	133	8.3	1756	1.8	3.6	51.6
37	6.8	1225	1.5	3.0	28.7	134	10.6	3061	1.4	6.1	59.9
38	9.3	2352	1.8	5.3	42.1	135	12.4	3149	2.1	5.3	87.7

39	11.7	3162	2.0	6.6	67.6	136	16.8	4980	1.8	8.9	137.2
40	16.5	3887	1.6	7.0	89.8	137	26.0	5499	2.0	9.6	220.6
41	23.9	5546	1.7	11.6	208.5	138	30.2	6312	2.1	12.8	316.3
42	29.5	5891	2.1	12.2	293.9	139	35.8	5941	2.3	12.1	380.4
43	34.9	5829	1.9	12.2	364.3	140	33.4	5604	1.5	11.8	311.6
44	33.2	5069	1.7	11.3	308.3	141	27.7	4353	1.5	10.1	218.3
45	27.4	4387	1.2	10.6	215.9	142	20.5	3523	1.1	8.4	138.9
46	22.1	3013	1.2	7.7	141.3	143	14.0	2664	0.6	7.1	76.5
47	10.9	2011	0.7	5.8	50.0	144	11.4	1302	1.7	3.2	29.9
48	9.0	1619	1.3	4.5	32.2	145	7.5	2067	1.1	4.9	39.9
ت	Y	X1	X2	X3	X4	ت	Y	X1	X2	X3	X4
49	4.3	2201	2.0	6.6	34.2	146	6.2	3021	1.5	6.1	44.8
50	6.8	2842	0.7	7.2	46.9	147	9.6	3375	1.8	6.4	78.0
51	14.1	3180	1.8	6.1	76.4	148	16.1	4517	1.9	7.6	121.0
52	21.3	4749	1.1	9.3	151.6	149	24.9	5025	2.0	9.3	224.4
53	26.1	5484	1.9	11.2	262.5	150	31.1	5473	1.6	11.5	332.3
54	30.4	5992	1.9	12.8	333.7	151	33.5	5898	2.1	12.4	392.9
55	34.7	5909	1.7	12.1	362.3	152	31.7	5501	2.1	11.9	352.3
56	33.6	5266	1.5	10.9	330.3	153	26.7	5134	1.0	10.3	242.8
57	28.0	4446	1.1	10.5	242.6	154	22.0	3227	1.0	7.8	137.9
58	22.1	3512	1.1	8.1	151.8	155	14.0	2514	0.4	7.0	59.7
59	13.8	1989	1.1	4.6	50.0	156	8.8	1672	0.7	3.8	23.7
60	7.6	1527	0.9	3.9	17.7	157	6.2	1794	1.3	3.5	24.6
61	5.1	2233	0.8	5.0	21.5	158	8.2	2982	1.2	6.1	53.7
62	8.2	2709	2.1	4.6	40.0	159	12.4	3602	1.8	6.8	94.2
63	13.4	4975	1.8	8.1	100.3	160	18.4	5075	1.7	8.6	136.2
64	16.8	5531	1.7	8.0	113.4	161	24.9	4968	1.9	10.1	241.8
65	24.7	5296	2.1	10.7	236.9	162	33.3	5679	2.3	12.3	391.5
66	30.6	7917	1.7	12.6	308.7	163	35.7	5473	1.8	12.1	426.3
67	35.3	5671	1.8	11.9	365.1	164	35.4	5399	1.3	11.6	390.8
68	32.9	6690	1.4	11.8	309.9	165	29.0	5433	1.5	10.5	284.8
69	28.3	5823	1.4	10.4	241.3	166	22.0	3641	1.2	9.1	197.1
70	22.0	4012	1.0	8.7	141.6	167	17.3	2712	0.6	6.8	92.9
71	15.2	2316	0.4	6.8	69.8	168	11.1	2069	0.7	5.8	49.4
72	8.6	2047	0.8	5.3	30.2	169	9.1	1915	1.0	5.7	46.1
73	6.8	1650	0.8	4.1	21.7	170	10.4	2812	1.3	6.7	61.7
74	7.5	2564	1.1	6.1	38.5	171	13.6	3598	2.0	7.2	107.7
75	13.1	2791	1.8	5.2	67.0	172	19.6	5652	1.9	9.5	185.4
76	19.0	4334	1.8	8.2	135.8	173	27.6	6429	2.7	10.8	361.9
77	23.3	4850	2.2	10.7	225.6	174	32.2	6661	1.8	11.5	380.6
78	32.0	6480	1.6	11.9	315.2	175	35.4	6711	2.5	11.3	434.5
79	34.5	5552	1.9	11.7	354.5	176	34.8	6175	2.1	10.7	397.0

80	33.6	6148	1.6	11.1	299.4	177	28.7	5116	1.2	10.7	272.0
81	28.4	5389	1.2	10.6	237.6	178	22.8	4055	0.8	8.3	161.6
82	22.3	4186	0.9	7.5	124.2	179	13.4	2574	0.5	6.8	76.8
83	15.0	2364	1.2	6.8	70.6	180	9.4	1915	0.6	4.5	38.3
84	7.7	1801	1.1	3.5	24.6	181	6.7	2053	1.9	4.3	31.9
85	3.7	1899	1.8	3.9	23.8	182	8.3	2897	1.3	6.7	65.4
86	5.8	2758	2.8	2.7	31.3	183	11.8	4572	1.7	7.7	110.9
87	9.8	3591	1.2	6.8	84.1	184	21.2	5054	2.8	7.3	216.5
88	16.4	4176	1.3	8.0	131.8	185	26.3	6482	2.1	9.8	294.2
89	21.9	4405	2.0	8.1	200.3	186	31.9	7335	2.3	12.5	375.6
90	29.0	5044	1.4	10.8	300.6	187	37.4	6562	2.4	11.3	464.1
91	32.2	5434	1.7	12.2	370.5	188	34.5	6261	2.3	11.4	403.3
92	33.2	5606	1.7	11.1	363.2	189	28.6	5307	1.8	10.1	275.3
93	27.7	4955	1.0	10.1	247.8	190	20.8	3192	1.5	8.4	158.3
94	21.0	4360	0.5	9.0	153.5	191	13.8	2618	1.1	6.9	72.8
95	12.4	2412	1.2	5.4	54.5	192	9.2	1992	2.0	3.5	25.1
96	6.6	1555	1.3	3.2	26.5						
97	5.6	2149	1.1	4.6	28.0						