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Reliability Estimation of the Lomax Distribution under Ranked Set Sampling (RSS) With Application

Musa Mohammed Musa[1] and Ban Ghanim Al-Ani[2]

[1,2]Department of Statistics and Informatics, College of Computer Science and Mathematics University of Mosul, Mosul, Iraq

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Correspondence:
Ban Ghanim Al-Ani
drbanalani@uomosul.edu.iq

Abstract

Sometimes the researcher faces a problem in obtaining data, or there may be difficulty in obtaining it due to cost, effort, or other reasons related to the time. In this case, sampling methods are used that ensure the researcher achieves his desired goal with a short time, effort, and cost, by using Ranked Set Sampling (RSS). In this paper, the reliability function of the Lomax Distribution was estimated under the (RSS) using four estimation methods, which are the Maximum Likelihood Estimators (MLE), the Maximum Product of Spacings (MPS), the Least Squares (LS) method, and Weighted Least Squares (WLS). The Monte Carlo simulation method was also used to determine the best method, and the best estimate was chosen using the Mean Square Error (MSE) criterion, and the results were applied in the theoretical aspect using the R-program, as the simulation results showed that the most efficient method among the methods used to estimate the reliability function of the Lomax distribution under (RSS) is the (MPS) method. The experimental aspect was applied to real data representing the times of the beginning of complete recovery (times of disease remission) in months for bladder cancer patients for a sample consisting of 96 patients drawn using the (RSS) method.

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1. Introduction

The most common approach to data collection uses the concept of simple random sampling (SRS) from the population. However, sometimes sampling is difficult because the cost of collecting data is expensive or requires a long time. Therefore, cost-optimal sampling methods have received more attention from statisticians, especially when measuring the characteristic of interest is expensive and requires more time to measure it. The idea of Ranked set sampling (RSS) was first proposed by Mclntyre (1951) through his effective attempts to find a more effective estimator to estimate the production of large pasture fields in Australia.

There have been several new developments of the idea introduced by McIntyre, which have made the method applicable to a much wider scope in areas such as environmental science, reliability and quality control. The (RSS) method has become an effective alternative to (SRS), as studies presented by many researchers have proven that it is more efficient by using some statistical criteria, including variance reducing of the estimator, and thus gives more accuracy when taking samples of a smaller size than in simple random sampling .

The (RSS) can be applied in many studies when the characteristic of interest is very difficult to measure (money, time, work, and organization) but the variable under study can be easily ranked even though it cannot be easily measured. Rankings may be made on the basis of visual inspection, prior information, or other approximate methods that do not require actual measurement. There are many researches and studies that dealt with the (RSS) method, including the work were the researcher A. Wolfe showed that the (RSS) is a statistical method for collecting data that gives a more efficient estimator than simple random sampling. He also explained how to obtain (RSS) and the basic difference. Between it and the (SRS) method (Wolfe, 2004). Hassan estimated the parameters of the exponential distribution based on (RSS) and used the Bayes method and the maximum likelihood estimator and it turned out that the Bayes estimate is the best (Hassan, 2013). A study was conducted by researchers (Al-Omari *et al.*) to estimate the reliability function when the distributions of both stress and force are independent and follow the exponential Pareto distribution, using the (MLS) method to estimate the reliability of stress strength under (SRS) and (RSS). The performance of the estimators was compared through a simulation study. The study revealed that stress strength reliability estimates under (RSS) are more efficient than (SRS) (Al-Omari, *et al*, 2020)

2. Material and methods

Ranked Set Sampling (RSS)

The concept (RSS) is a statistical method to collect data by reducing the sample size and obtaining real measurements with the shortest time and least cost, that is, through which we will obtain measurements that have the most luck to represent the population, which is an alternative to simple random sampling (SRS). The steps for selecting RSS are as follows (Sabry & Al-Metwally, 2021):

1. We select (m) sets randomly from the population under study, each set of size (m).
2. The elements of each set in step (1) are arranged according to a predetermined property, such as ascending or descending order.
3. After the arrangement, we take the smallest ordered unit from the first set and the smallest second ordered unit from the second set, and the selection process continues until the largest ordered unit is selected from the last set. The analysis includes only the specific units (m) that enter into the analysis only, and we can reverse the process by choosing the largest unit from the first set and so on to get a new group of size (m), which is called the ranked set sample.
4. Repeat steps (1-3) for (c) of cycles until we get a sample of size n = mc. where $y_{(ii)k}$ represents the ordered unit of sequence i (i=1,2,…,m) in the i-th set in the cycle k (k=1,2,…,c) and represents a single from the sample of the ordered sets with a sample size n=mc

$$\text{Step1:}\begin{bmatrix} y_{11} < & y_{12} < & \cdots & < y_{1m} \\ y_{21} < & y_{22} < & \cdots & < y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} < & y_{m2} < & \cdots & < y_{mm} \end{bmatrix}$$

$$\text{Step2:} [y_{11}, y_{22}, \cdots, y_{mm}]$$

For ease, the second step will be written in the following format

$$\Longrightarrow [y_{(1)}, y_{(2)}, \cdots, y_{(m)}]$$

Repeating the operation to c of the cycles gives us:

$$\Longrightarrow \text{Step3} \begin{bmatrix} y_{(1)1} & y_{(2)1} & \cdots & y_{(m)1} \\ y_{(1)2} & y_{(2)2} & \cdots & y_{(m)2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{(1)c} & y_{(2)c} & \cdots & y_{(m)c} \end{bmatrix}_{m*c}$$

3- Lomax Distribution

The Lomax distribution is known as the Pareto distribution of the second type. It is useful for modeling and analyzing survival data in medical, biological, engineering, etc. sciences. It has received a great deal of attention from statisticians due to its use in the study of reliability and life. The first to use Lomax distribution was the scientist Lomax in (1954). Let the random variable y follow the Lomax distribution with the parameters β and θ, where θ is a shape parameter and β is a scale parameter, the probability density function (pdf), the cumulative function (cdf) and the reliability function of the variable y respectively are as follows:

$$f(y) = \frac{\theta}{\beta}\left(1+\frac{y}{\beta}\right)^{-(\theta+1)} \quad y>0 \quad ,\theta,\beta>0$$

$$F(y) = 1-\left(1+\frac{y}{\beta}\right)^{-\theta}$$

$$R(t) = P(T>t) = \left(1+\frac{t}{\beta}\right)^{-\theta} \; ; t>0$$

4- Probability Density Function for (RSS)

Let $y_{(11)1}, y_{(22)1}, \ldots, y_{(mm)c}$ be a random sample drawn by the (RSS) method obtained from cycles (c) of size (m) where $y_{(ii)k}$ are independent random variables, and represent the ordered statistics of size mc. It has the following probability density function:

$$f\left(y_{(ii)k}\right) = \frac{m!}{(i-1)!\,(m-i)!} f\left(y_{(ii)k}\right)\{F(y_{(ii)k})\}^{i-1}\{1 - F\left(y_{(ii)k}\right)\}^{m-i}$$
$$i = 1,2,\ldots,m\,;\, k = 1,2,\ldots c \quad (1)$$

5- Estimation Methods

5-1 Maximum likelihood Estimators under RSS

This method is based on the concept of the likelihood function, let $y_1, y_2, \ldots, y_n$ represent the measurements of a random sample drawn from a population with a probability density function f(y,θ) ; θ ∈ Ω, the likelihood function is defined as the joint distribution of those measurements. The principle of the likelihood method can be established in finding estimates of the parameters of probability distributions, which makes the likelihood function to its maximum end. Therefore, the likelihood function of the Lomax distribution under (RSS) is as follows (Aziz & Shaaban, 2021) , (Sabry, et al, 2019):

$$L\left(\theta;\beta\middle|y_{(ii)k}\right) = \prod_{i=1}^{m}\prod_{c=1}^{k} f\left(y_{(ii)k}\right) \quad (2)$$

We substitute the probability density function of (RSS) into equation (1) as follows:

$$L(\theta;\beta|y_{(ii)k}) = \prod_{i=1}^{m}\prod_{c=1}^{k}\frac{m!}{(i-1)!\,(m-1)!}\, f\left(y_{(ii)k}\right)\{F(y_{(ii)k})\}^{i-1}\{1 - F\left(y_{(ii)k}\right)\}^{m-i} \quad (3)$$

$$L\left(\theta;\beta|y_{(ii)k}\right) = \frac{m!}{(i-1)!\,(m-i)!}\frac{\theta}{\beta}(1+\frac{y_{(ii)k}}{\beta})^{-(\theta+1)}\{1\text{-}(1+\frac{y_{(ii)k}}{\beta})^{-\theta}\}^{i-1}\{(1+\frac{y_{(ii)k}}{\beta})^{-\theta}\}^{m-i} \quad (4)$$

By taking ln for equation (4), we get the following equation:

$$lnL(\theta,\beta) = \prod_{i=1}^{m}\prod_{k=1}^{c}\frac{m!}{(i\text{-}1)!\,(m\text{-}1)!} + km\,ln\,\theta\text{-}km\,ln\,\beta + \sum_{i=1}^{m}\sum_{k=1}^{c}\{\theta(i\text{-}m\text{-}1)\text{-}1\}\ln\left\{1+\frac{y_{(i)j}}{\beta}\right\} + \sum_{i=1}^{m}\sum_{k=1}^{c}(i\text{-}1)\ln\left\{1\text{-}(1+\frac{y_{(i)j}}{\beta})^{-\theta}\right\} \quad (5)$$

We differentiate equation (5) with respect to θ and then equalize it to zero we get:

$$\frac{km}{\theta} + \sum_{i=1}^{m}\sum_{c=1}^{k}(i-m-1)\ln\left\{1+\frac{y_{(ii)k}}{\beta}\right\} - \sum_{i=1}^{m}\sum_{c=1}^{k}\left\{\frac{(i-1)(1+\frac{y_{(ii)k}}{\beta})^{-\theta}\ln(1+\frac{y_{(ii)k}}{\beta})^{-1}}{1-(1+\frac{y_{(ii)k}}{\beta})^{-\theta}}\right\} = 0 \quad (6)$$

We differentiate equation (5) with respect to β and then equalize it to zero we get:

$$\sum_{i=1}^{m}\sum_{c=1}^{k}\left\{\frac{\{\theta(i\text{-}m\text{-}1)\text{-}1\}\left(\frac{\text{-}y_{(ii)k}}{\beta^2}\right)}{\left(1+\frac{y_{(ii)k}}{\beta}\right)}\right\} + \sum_{i=1}^{m}\sum_{c=1}^{k}\left\{\frac{\theta(i\text{-}1)(1+\frac{y_{(ii)k}}{\beta})^{-(\theta+1)}(\text{-}\frac{y_{(ii)k}}{\beta^2})}{1\text{-}(1+\frac{y_{(ii)k}}{\beta})^{-\theta}}\right\}\text{-}\frac{km}{\beta} = 0 \quad (7)$$

The equations (6 & 7) cannot be solved by ordinary mathematical methods, so they will be solved numerically using Newton-Raphson's iterative method to obtain the estimators $\hat{\theta}_{LRSS}$,$\hat{\beta}_{LRSS}$. Therefore, the estimator of the reliability function by the maximum likelihood method under (RSS) of the Lomax distribution is as follows:

$$R(t) = P(T > t) = \left(1 + \frac{t}{\hat{\beta}_{LRSS}}\right)^{-\hat{\theta}_{LRSS}} \;; t > 0 \quad (8)$$

5-2 Maximum Product of Spacing Estimator (MPS) method

If $y_1, y_2, ..., y_n$ is a random sample arranged and spaced on a regular form between its individuals and taken from a population that follows the probability distribution that has a function of a probability density function f(y,θ,β) and a cumulative function F(y), then the highest product of the spacing estimators we get from maximizing the geometric mean of the distances, and the estimate in this method increases the spacing to the maximum extent of the geometric mean Q(θ;β|y), this can be illustrated as follows (Al-Metwally & Al-Mongy, 2019), (Al-Omari, *et al*, 2021):

$$Q(\theta;\beta|y) = \left\{\prod_{i=1}^{n+1} Z_i(\theta;\beta|y)\right\}^{\frac{1}{n+1}} \tag{9}$$

where

$$Z_i(\theta,\beta|y) = \begin{cases} Z_1 = F(y_1) \\ Z_i = F(x_i) - F(y_{i-1}) & i = 2,3,\dots,n \\ Z_{n+1} = 1 - F(y_n) \end{cases}$$

and $\sum z_i = 1$, such that $F(y_{(0)}) = 0$, $F(y_{(n+1)}) = 1$, so

$$Q(\theta;\beta|y) = \left\{\prod_{i=1}^{n+1} F\left(y_{(i:n)}/\theta,\beta\right) - F\left(y_{(i-1:n)}/\theta,\beta\right)\right\}^{\frac{1}{n+1}} \tag{10}$$

After substituting the cdf of Lomax distribution and taking the Ln, we get:

$$\ln Q(\theta;\beta) = \frac{1}{n+1}\sum_{i=1}^{n+1}\left\{1-(1+\frac{y_{(i:n)}}{\beta})^{-\theta}\right\} - \left\{1-(1+\frac{y_{(i-1:n)}}{\beta})^{-\theta}\right\}^{\frac{1}{n+1}} \tag{11}$$

We derive equation (11) with respect to θ and then equalize it to zero

$$\frac{\partial lnQ(\theta;\beta)}{\partial\theta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\left\{\frac{(-(1+\frac{y_{i:n}}{\beta})^{-\theta}\ln(1+\frac{y_{(i:n)}}{\beta})^{-1} + (1+\frac{y_{(i-1:n)}}{\beta})^{-\theta}\ln(1+\frac{y_{(i-1:n)}}{\beta})^{-1})}{\{1-(1+\frac{y_{(i:n)}}{\beta})^{-\theta}\}-\{1-(1+\frac{y_{(i-1:n)}}{\beta})^{-\theta}\}}\right\}$$

$$=0 \tag{12}$$

We derive equation (11) with respect to β and then equalize it to zero

$$\frac{\partial lnQ(\beta;\lambda)}{\partial\beta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\left\{\frac{(\frac{\theta y}{\beta^2}(1+\frac{y_{(i:n)}}{\beta})^{-(\theta+1)} - \frac{\theta y}{\beta^2}(1+\frac{y_{(i-1:n)}}{\beta})^{-(\theta+1)})}{\{1-(1+\frac{y_{(i:n)}}{\beta})^{-\theta}\}-\{1-(1+\frac{y_{(i-1:n)}}{\beta})^{-\theta}\}}\right\} = 0 \tag{13}$$

The equations (12 & 13) cannot be solved by ordinary mathematical methods, so they will be solved numerically using Newton-Raphson's iterative method to obtain the estimations $\hat{\theta}_{MPS(RSS)}$,$\hat{\beta}_{MPS(RSS)}$. Therefore, the estimator of the reliability function by the maximum likelihood method under (RSS) of the Lomax distribution is as follows:

$$R(t) = P(T > t) = \left(1 + \frac{t}{\hat{\beta}_{MPS(RSS)}}\right)^{-\hat{\theta}_{MPS(RSS)}} \quad ; t > 0 \tag{14}$$

5-3 Least Squares Method

Swain in 1988 was the first who used to estimate beta distribution parameters based on probability theory that indicates that $F(y(i:n) \sim Beta(i, n-i+1)$ where $F(y(i:n)$ is the cumulative distribution function and $y_{(i:n)}$ is i-th ordered statistics for the random sample $(y_1, y_2, \dots, y_n)$. It is also one of the important methods in estimation processes, as this method depends on reducing the set of error squares that can formulate its equation as follows (Al-Omari, *et al*, 2021), (Taconeli & Bonat, 2019):

$$\Omega(\theta;\beta|y) = \sum_{i=1}^{n}\{F(y_{i:n}/\theta,\beta) - \frac{i}{n+1}\}^2 \tag{15}$$

We substitute the cdf function of the Lomax distribution into equation (15)

$$\Rightarrow \Omega(\theta;\beta|y_{i:n}) = \sum_{i=1}^{n}\{1-(1+\frac{y_{i:n}}{\beta})^{-\theta}-\frac{i}{n+1}\}^2 \qquad (16)$$

We derive equation (16) with respect to θ and then equalize it to zero

$$\frac{\partial\Omega(\theta;\beta|y_{i:n})}{\partial\theta} = -2\sum_{i=1}^{n}\left\{1-(1+\frac{y_{i:n}}{\beta})^{-\theta}-\frac{i}{n+1}\right\}*\{(1+\frac{y_{i:n}}{\beta})^{-\theta}\ln(1+\frac{y_{i:n}}{\beta})^{-1}\right\} = 0 \qquad (17)$$

We derive equation (17) with respect to β and then equalize it to zero

$$\frac{\partial\Omega(\theta;\beta|y_{i:n})}{\partial\beta} = \frac{2}{\beta^2}\sum_{i=1}^{n}\left\{1-(1+\frac{y_{i:n}}{\beta})^{-\theta}-\frac{i}{n+1}\right\}*\{\theta(1+\frac{y_{i:n}}{\beta})^{-(\theta+1)}(y_{i:n})\right\} = 0 \qquad (18)$$

The equations (17 & 18) cannot be solved by ordinary mathematical methods, so they will be solved numerically using Newton-Raphson's iterative method to obtain the estimations $\hat{\theta}_{ols(RSS)}$,$\hat{\beta}_{ols(RSS)}$. Therefore, the estimator of the reliability function by the maximum likelihood method under (RSS) of the Lomax distribution is as follows:

$$R(t) = P(T>t) = \left(1+\frac{t}{\hat{\beta}_{ols(RSS)}}\right)^{-\hat{\theta}_{ols(RSS)}} ; t>0 \qquad (19)$$

5-4 Weighted Least Squares Method

The weighted least squares (WLS) method is a basic method of estimation, which contains the weight factor (w_i) and to distinguish between this method and the method of ordinary least squares based on the concept of reducing the sum of the squares of error and its shape as much as possible. Suppose y_i are ordered statistics taken from a random sample of size n resulting from a continuous probability distribution. The following formula can be followed for the (WLS) method:

$$K(\theta;\beta|y) = \sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)} * \left\{F(y_i)-\frac{i}{n+1}\right\}^2 \qquad (20)$$

We substitute the cdf function of the Lomax distribution into equation (20)

$$K(\theta;\beta|y_{i:n}) = \sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)} * \left\{1-(1+\frac{y_{i:n}}{\beta})^{-\theta}-\frac{i}{n+1}\right\}^2 \qquad (21)$$

We derive equation (21) with respect to θ and then equalize it to zero

$$\frac{\partial K(\theta;\beta|y_{i:n})}{\partial\theta} = \frac{-2(n+1)^2(n+2)}{i(n+i-1)}\sum_{i=1}^{n}\left\{\left(1+\frac{y_{i:n}}{\beta}\right)^{-\theta}\ln\left(1+\frac{y_{i:n}}{\beta}\right)^{-1}\left(1-\left(1+\frac{y_{i:n}}{\beta}\right)^{-\theta}\right)-\frac{i}{n+1}\right\}$$
$$= 0 \qquad (22)$$

We derive equation (21) with respect to β and then equalize it to zero

$$\frac{\partial K(\theta;\beta|y)}{\partial\beta} = \frac{2\theta}{\beta^2}\frac{(n+1)^2(n+2)}{i(n+i-1)}\sum_{i=1}^{n}\left\{(1+\frac{y_{i:n}}{\beta})^{-(\theta+1)}\{1-(1+\frac{y_{i:n}}{\beta})^{-\theta}-\frac{i}{n+1}\}(y_{i:n})\right\} = 0 \qquad (23)$$

The equations (22 & 23) cannot be solved by ordinary mathematical methods, so they will be solved numerically using Newton-Raphson's iterative method to obtain the estimations $\hat{\theta}_{WLS(RSS)}$,$\hat{\beta}_{WLS(RSS)}$. Therefore, the estimator of the reliability function by the maximum likelihood method under (RSS) of the Lomax distribution is as follows:

$$R(t) = P(T>t) = \left(1+\frac{t}{\hat{\beta}_{WLS(RSS)}}\right)^{-\hat{\theta}_{WLS(RSS)}} ; t>0 \qquad (24)$$

6. Statistical Analysis

. Experimental Side:

6-1 Simulation experiment stages:

The simulation program was written using the statistical programming language R. The program includes four basic stages to estimate the parameters and reliability function of the Lomax distribution, as follows:

The first stage: Specifying default values

The values were chosen as follows:

1. The default values were chosen for the two Lomax distribution parameters, and three models were formed, as follows:

Table (1): Default values for Lomax distribution parameters

Model	θ	β	
1	0.5	0.5	θ = β
2	0.8	0.5	θ > β
3	0.5	1.5	θ < β

2. Different sample sizes were chosen (12, 24, 36, 54,96), as follows:

Table (2): The used Sample sizes

Sample size	) m(group size	) k(cycles number
12	3	4
	4	3
	6	2
24	3	8
	6	4
	4	6
36	4	9
	6	6
	3	12
54	9	6
	6	9
	3	18
96	8	12
	12	8
	6	16

3. Each experiment was repeated 1000 times.

The second stage: generating data

This is a very important stage on which the subsequent steps depend, as the random variable that follows the Lomax distribution is generated by applying the inverse transformation method, as follows :

$$F(t) = 1 - [1 + \frac{t}{\beta}]^{-\theta}$$

$$u = 1 - [1 + \frac{t}{\beta}]^{-\theta}$$

Taking the ln

$$\ln\left(1 + \frac{t}{\beta}\right) = -\frac{\ln(1-u)}{\theta}$$

$$t = \beta\left[e^{-\frac{\ln(1-u)}{\theta}} - 1\right]$$

$$t = \beta e^{-\frac{\ln(1-u)}{\theta}} - \beta$$

The value of u is replaced by a generated value that follows a uniform distribution within the interval [0 , 1].

The third stage: The estimation
At this stage, the estimation process for the reliability function of the Lomax distribution is performed using the estimation methods mentioned in the theoretical aspect under the (RSS) method.

The fourth stage: the comparison stage between methods
To compare different estimation methods for the reliability function and find the best estimators, statistical criteria must be used, such as the MSE, since the method that has the lowest MSE value is considered better, such that:

$$MSE\left(\hat{R}(t)\right) = \frac{1}{L}\sum_{i=1}^{L}\left[\hat{R}_i(t) - R_i(t)\right]^2$$

since:
L: represents the number of repetitions for each experiment, which is equal to (1000) .
$\hat{R}_i(t)$: is an estimation of $R_i(t)$ according to the used estimation method.

6-2 Experimental results using the (RSS) method.

To apply estimation methods for the reliability function of the Lomax distribution and determine the best method, which will be used in estimating the reliability function for real data in the applied aspect using the R program, the simulation results were presented in tables that included a comparison between the estimation methods for the reliability function under the (RSS) method.

Based on the equations (6, 7 & 8) for the (RSS) method, the equations (12, 13 & 24) for the MPS method, the equations (17, 18 & 19) for the OLS method, and the equations (22, 23 & 24) for the WLS method, the MSE criteria values were found for each estimation, and the results were as in the following tables.

Table (3). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 0.5 and a sample size of n = 12

m	k	t	Values					MSE				Best
			Real	**MLE**	**MPS**	**OLS**	**WLS**	**MLE**	**MPS**	**OLS**	**WLS**	
3	4	0.05	0.9535	0.9539	0.9429	0.9454	0.9470	0.0011	0.0011	0.0010	0.001	MPS
		0.5	0.7071	0.7278	0.6983	0.7026	0.7057	0.0152	0.0106	0.0111	0.0111	
		1.5	0.5	0.5173	0.5088	0.5052	0.5076	0.0226	0.0134	0.0152	0.0153	
		3.5	0.3536	0.3498	0.3711	0.3603	0.3615	0.0215	0.0125	0.0150	0.0146	
		5	0.3015	0.2887	0.3211	0.3085	0.3090	0.0196	0.0116	0.0141	0.0135	
		ALL	-	-	-	-	-	0.0160	0.0098	0.0113	0.0111	
6	2	0.05	0.9535	0.9507	0.9416	0.9437	0.9455	0.0020	0.0012	0.0012	0.0011	MPS
		0.5	0.7071	0.7248	0.6948	0.6985	0.7021	0.0179	0.0106	0.0109	0.0109	
		1.5	0.5	0.5163	0.5047	0.5029	0.5049	0.0263	0.0127	0.0143	0.0143	
		3.5	0.3536	0.3493	0.3669	0.3600	0.3600	0.0250	0.0118	0.0141	0.0137	
		5	0.3015	0.2881	0.3170	0.3088	0.3080	0.0226	0.0111	0.0133	0.0129	
		ALL	-	-	-	-	-	0.0188	0.0095	0.0108	0.0106	

Table (4). Real and estimated reliability values and their associated MSE values when θ = 0.8, β = 0.5 and a sample size of n = 12

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
3	4	0.05	0.927	0.9297	0.9212	0.9231	0.9246	0.0027	0.0018	0.0016	0.002	WLS
		0.5	0.574	0.6085	0.5969	0.5936	0.5984	0.0179	0.0127	0.0126	0.013	
		1.5	0.33	0.3373	0.3594	0.3523	0.354	0.021	0.0123	0.0131	0.012	
		3.5	0.189	0.1698	0.2104	0.2057	0.2015	0.016	0.0094	0.01	0.009	
		5	0.147	0.1242	0.1653	0.1619	0.1558	0.0128	0.0079	0.0084	0.007	
		ALL	-	-	-	-	-	0.0141	0.0088	0.0092	0.009	
6	2	0.05	0.927	0.9251	0.9151	0.9182	0.9196	0.0034	0.0021	0.0024	0.002	MPS
		0.5	0.574	0.5908	0.5771	0.5838	0.5837	0.0319	0.0128	0.0143	0.014	
		1.5	0.33	0.3372	0.3543	0.3481	0.3482	0.0365	0.013	0.0146	0.015	
		3.5	0.189	0.1842	0.2197	0.2059	0.207	0.0232	0.01	0.0112	0.011	
		5	0.147	0.1383	0.1774	0.1629	0.1639	0.0171	0.0088	0.0095	0.009	
		ALL	-	-	-	-	-	0.0224	0.0093	0.0104	0.01	

Table (5). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 1.5 and a sample size of n = 12

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
3	4	0.05	0.97411	0.97645	0.97128	0.9719	0.9725	0.0001687	0.000177	0.00017	0.0002	OLS
		0.5	0.79442	0.81217	0.78903	0.7912	0.79375	0.0070002	0.00556	0.00558	0.0056	
		1.5	0.57435	0.5977	0.58146	0.5821	0.58445	0.0175338	0.011001	0.01124	0.0119	
		3.5	0.38168	0.3882	0.39586	0.3929	0.39379	0.0203828	0.010728	0.01024	0.0111	
		5	0.30942	0.30472	0.32336	0.3184	0.31852	0.0192114	0.009614	0.00871	0.0093	
		ALL	-	-	-	-	-	0.0128594	0.007416	0.00719	0.0076	
6	2	0.05	0.97411	0.97268	0.96893	0.9702	0.97116	0.0002233	0.000124	9.7E-05	9E-05	MPS
		0.5	0.79442	0.78811	0.76931	0.7721	0.77806	0.0064329	0.00357	0.00316	0.0028	
		1.5	0.57435	0.55391	0.54535	0.5376	0.54696	0.0151262	0.006968	0.00745	0.0064	
		3.5	0.38168	0.33636	0.35768	0.3356	0.3455	0.0173254	0.00803	0.01103	0.0093	
		5	0.30942	0.25478	0.28917	0.2631	0.27225	0.0156661	0.007782	0.01175	0.01	
		ALL	-	-	-	-	-	0.0109548	0.005295	0.0067	0.0057	

Table (6). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 0.5 and a sample size of n = 24

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
3	8	0.05	0.9535	0.9493	0.94444	0.9463	0.94798	0.0011	0.00065	0.00062	0.00054	MPS
		0.5	0.7071	0.7097	0.69458	0.6981	0.70131	0.0096	0.00602	0.00634	0.00614	
		1.5	0.5	0.5042	0.50127	0.4997	0.501	0.0118	0.0067	0.00754	0.00743	
		3.5	0.3536	0.3497	0.36384	0.3572	0.35659	0.0101	0.00598	0.00704	0.00676	
		5	0.3015	0.2939	0.31441	0.3062	0.30489	0.0092	0.00561	0.00668	0.00632	
		ALL	-	-	-	-	-	0.0084	0.00499	0.00564	0.00544	
6	4	0.05	0.9535	0.952	0.94542	0.9477	0.94904	0.0008	0.0007	0.00068	0.00061	MPS
		0.5	0.7071	0.7167	0.69892	0.7039	0.70645	0.0087	0.00621	0.00647	0.00638	
		1.5	0.5	0.5095	0.50566	0.5045	0.5059	0.0111	0.00701	0.00775	0.00772	
		3.5	0.3536	0.3527	0.36698	0.3594	0.35945	0.01	0.00624	0.00727	0.00702	
		5	0.3015	0.2961	0.3169	0.3073	0.30681	0.0091	0.00582	0.00691	0.00656	
		ALL	-	-	-	-	-	0.0079	0.0052	0.00581	0.00566	

Table (7). Real and estimated reliability values and their associated MSE values when θ = 0.8, β = 0.5 and a sample size of n = 24

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
3	8	0.05	0.9266	0.93	0.92028	0.9251	0.92592	0.0012	0.00099	0.00084	0.00081	MPS
		0.5	0.5743	0.5975	0.57493	0.5783	0.5804	0.0137	0.00855	0.00899	0.00886	
		1.5	0.3299	0.341	0.3409	0.3311	0.33266	0.0124	0.00776	0.00887	0.00871	
		3.5	0.1895	0.1816	0.2023	0.1862	0.18714	0.0074	0.00495	0.00561	0.00545	
		5	0.1469	0.1345	0.15946	0.1428	0.14354	0.0055	0.00382	0.00415	0.00399	
		ALL	-	-	-	-	-	0.008	0.00521	0.00569	0.00556	
6	4	0.05	0.9266	0.9244	0.91762	0.9213	0.92254	0.0016	0.00103	0.001	0.00092	MPS
		0.5	0.5743	0.5802	0.56947	0.575	0.57585	0.0099	0.00607	0.00677	0.0069	
		1.5	0.3299	0.3237	0.33903	0.3332	0.33334	0.0105	0.0064	0.00728	0.00705	
		3.5	0.1895	0.1763	0.20539	0.193	0.19208	0.0082	0.00535	0.00601	0.00543	
		5	0.1469	0.1346	0.16439	0.1516	0.15004	0.0068	0.00467	0.00512	0.00455	
		ALL	-	-	-	-	-	0.0074	0.0047	0.00523	0.00497	

Table (8). Real and estimated reliability values and their associated MSE values when θ = 0.8, β = 1.5 and a sample size of n = 24

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
3	8	0.05	0.9837	0.9847	0.9801	0.982	0.9818	9.05E-05	0.00014	7.6E-05	9.5E-05	MPS
		0.5	0.866	0.8772	0.8529	0.8604	0.86	0.003629	0.00362	0.00251	0.003	
		1.5	0.7071	0.7299	0.6997	0.7069	0.7071	0.009544	0.00625	0.00539	0.00608	
		3.5	0.5477	0.5707	0.5502	0.5547	0.5547	0.011957	0.00604	0.00622	0.0065	
		5	0.4804	0.4991	0.4866	0.4899	0.4893	0.011595	0.00562	0.0061	0.00617	
		ALL	-	-	-	-	-	0.007363	0.00434	0.00406	0.00437	
6	4	0.05	0.9837	0.9849	0.9828	0.9832	0.9838	4.76E-05	2.5E-05	2.4E-05	2.2E-05	MPS
		0.5	0.866	0.878	0.8641	0.8663	0.8698	0.001848	0.00095	0.00093	0.00094	
		1.5	0.7071	0.7304	0.713	0.714	0.7188	0.004533	0.00252	0.00262	0.00277	
		3.5	0.5477	0.5694	0.5643	0.5618	0.566	0.006011	0.00394	0.00424	0.00442	
		5	0.4804	0.4969	0.5013	0.4968	0.5005	0.006591	0.0045	0.00486	0.00496	
		ALL	-	-	-	-	-	0.003806	0.00239	0.00254	0.00262	

Table (9). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 0.5 and a sample size of n = 36

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
9	4	0.05	0.9535	0.9524	0.9465	0.9481	0.9494	0.0005	0.0005	0.0004	0.0004	MPS
		0.5	0.7071	0.7131	0.6966	0.7002	0.7027	0.0062	0.0044	0.0045	0.0044	
		1.5	0.5	0.5051	0.4999	0.4994	0.5002	0.0073	0.0046	0.005	0.005	
		3.5	0.3536	0.3505	0.3604	0.3556	0.3547	0.0061	0.0041	0.0045	0.0044	
		5	0.3015	0.2949	0.3105	0.3042	0.3027	0.0057	0.0038	0.0043	0.0041	
		ALL	-	-	-	-	-	0.0051	0.0035	0.0037	0.0036	
6	6	0.05	0.9535	0.9524	0.9473	0.949	0.9503	0.0004	0.0004	0.0004	0.0004	MPS
		0.5	0.7071	0.7121	0.699	0.7022	0.7052	0.005	0.0041	0.0042	0.0042	
		1.5	0.5	0.505	0.5024	0.5011	0.5025	0.006	0.0044	0.0048	0.0048	
		3.5	0.3536	0.3525	0.3626	0.3569	0.3565	0.0051	0.0038	0.0043	0.0041	
		5	0.3015	0.2978	0.3125	0.3053	0.3042	0.0047	0.0035	0.004	0.0038	
		ALL	-	-	-	-	-	0.0042	0.0032	0.0035	0.0035	

Table (10). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 1.5 and a sample size of

n = 36

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
9	4	0.05	0.9837	0.9862	0.9839	0.9849	0.9851	4.7E-5	3.9E-5	3.8E-5	3.4E-5	MPS
		0.5	0.866	0.886	0.8719	0.8777	0.8786	0.0022	0.0015	0.0016	0.0015	
		1.5	0.7071	0.7436	0.7241	0.7317	0.732	0.0067	0.0039	0.0047	0.0044	
		3.5	0.5477	0.5861	0.573	0.5772	0.5766	0.009	0.0052	0.0063	0.0059	
		5	0.4804	0.5144	0.5074	0.5087	0.5077	0.0089	0.0053	0.0063	0.006	
		ALL	-	-	-	-	-	0.0054	0.0032	0.0038	0.0036	
6	6	0.05	0.9837	0.9829	0.9804	0.9808	0.9815	5E-05	7E-05	7E-05	6E-05	MPS
		0.5	0.866	0.8646	0.8514	0.8542	0.8573	0.002	0.0022	0.0019	0.0019	
		1.5	0.7071	0.7107	0.6955	0.6999	0.7023	0.0053	0.0045	0.0042	0.0043	
		3.5	0.5477	0.5547	0.5466	0.5508	0.5506	0.007	0.0051	0.0054	0.0055	
		5	0.4804	0.4873	0.4841	0.488	0.4863	0.0069	0.005	0.0056	0.0056	
		ALL	-	-	-	-	-	0.0042	0.0034	0.0034	0.0034	

Table (11). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 0.5 and a sample size of n = 54

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
9	6	0.05	0.9535	0.9526	0.9482	0.9495	0.951	0.0004	0.0003	0.0003	0.00029	MPS
		0.5	0.7071	0.7112	0.6993	0.7024	0.704	0.0038	0.0033	0.0032	0.00314	
		1.5	0.5	0.5031	0.5005	0.4995	0.5	0.0036	0.003	0.003	0.00309	
		3.5	0.3536	0.3507	0.3591	0.3541	0.354	0.0026	0.0021	0.0024	0.00234	
		5	0.3015	0.2962	0.3083	0.3022	0.301	0.0023	0.0018	0.0023	0.00209	
		ALL	-	-	-	-	-	0.0025	0.0021	0.0022	0.00219	
3	18	0.05	0.9535	0.9498	0.946	0.9472	0.948	0.0007	0.0005	0.0004	0.00039	MPS
		0.5	0.7071	0.7067	0.6947	0.6968	0.7	0.0051	0.0038	0.0036	0.0037	
		1.5	0.5	0.5011	0.4965	0.496	0.497	0.0055	0.0035	0.0036	0.00365	
		3.5	0.3536	0.3516	0.3565	0.3533	0.352	0.0048	0.0027	0.0032	0.00301	
		5	0.3015	0.2984	0.3065	0.3024	0.301	0.0045	0.0025	0.0031	0.00278	
		ALL	-	-	-	-	-	0.0041	0.0026	0.0028	0.00271	

Table (12). Real and estimated reliability values and their associated MSE values when θ = 0.8, β = 0.5 and a sample size of n = 54

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
9	6	0.05	0.9266	0.9269	0.9229	0.9237	0.925	0.0005	0.0004	0.0005	0.00039	MPS
		0.5	0.5743	0.5793	0.5756	0.5777	0.578	0.0043	0.0028	0.0032	0.0031	
		1.5	0.3299	0.3274	0.3393	0.3379	0.336	0.0036	0.0024	0.0028	0.0027	
		3.5	0.1895	0.1808	0.2014	0.1977	0.194	0.0026	0.0018	0.002	0.00183	
		5	0.1469	0.1376	0.1589	0.155	0.152	0.0022	0.0015	0.0017	0.0015	
		ALL	-	-	-	-	-	0.0026	0.0018	0.002	0.0019	
3	18	0.05	0.9266	0.9259	0.9183	0.919	0.92	0.0007	0.0006	0.0006	0.00054	WLS
		0.5	0.5743	0.5813	0.565	0.5663	0.567	0.0058	0.0035	0.0037	0.00359	
		1.5	0.3299	0.3342	0.3341	0.3324	0.332	0.0059	0.0029	0.003	0.003	
		3.5	0.1895	0.1907	0.2008	0.1975	0.196	0.0045	0.0023	0.0022	0.00223	
		5	0.1469	0.1479	0.1597	0.1561	0.154	0.0037	0.002	0.0019	0.0019	
		ALL	-	-	-	-	-	0.0041	0.00225	0.0023	0.002252	

Table (13). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 1.5 and a sample size of n = 54

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
9	6	0.05	0.9837	0.9878	0.9856	0.9856	0.986	3E-05	2E-05	2E-05	2.4E-05	MPS
		0.5	0.866	0.8955	0.8809	0.8812	0.883	0.0017	0.001	0.0011	0.00116	
		1.5	0.7071	0.7575	0.7352	0.7363	0.739	0.005	0.0029	0.0033	0.0034	
		3.5	0.5477	0.6005	0.5813	0.5826	0.584	0.0062	0.0038	0.0041	0.00425	
		5	0.4804	0.5284	0.5136	0.5146	0.515	0.0057	0.0037	0.0038	0.00397	
		ALL	-	-	-	-	-	0.0037	0.0023	0.0025	0.00256	
3	18	0.05	0.9837	0.9839	0.9811	0.9815	0.982	4E-05	3E-05	3E-05	2.1E-05	MPS
		0.5	0.866	0.8696	0.8521	0.8543	0.857	0.0017	0.001	0.0009	0.00085	
		1.5	0.7071	0.7155	0.6923	0.6942	0.697	0.0048	0.0022	0.0021	0.00208	
		3.5	0.5477	0.556	0.5401	0.5402	0.542	0.0066	0.0025	0.0026	0.00255	
		5	0.4804	0.4865	0.4769	0.476	0.478	0.0067	0.0024	0.0026	0.00255	
		ALL	-	-	-	-	-	0.004	0.0016	0.0017	0.00161	

Table (14). Real and estimated reliability values and their associated MSE values when θ = 0.8, β = 0.5 and a sample size of n = 96

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
12	8	0.05	0.9535	0.952	0.9489	0.9498	0.9504	0.00018	0.00018	0.000159	0.00016	MPS
		0.5	0.7071	0.7055	0.6982	0.6992	0.701	0.002	0.00177	0.001752	0.00178	
		1.5	0.5	0.4986	0.4978	0.4958	0.497	0.00216	0.00173	0.001966	0.00189	
		3.5	0.3536	0.3511	0.3571	0.3527	0.3529	0.00192	0.00148	0.001889	0.00166	
		5	0.3015	0.2987	0.3069	0.3019	0.3017	0.00181	0.0014	0.001842	0.00157	
		ALL	-	-	-	-	-	0.00161	0.00131	0.001521	0.00141	
16	6	0.05	0.9535	0.9547	0.9515	0.9524	0.9531	0.00011	0.00013	0.000122	0.00012	MPS
		0.5	0.7071	0.7157	0.7077	0.7094	0.7112	0.00164	0.00145	0.001465	0.00149	
		1.5	0.5	0.5094	0.5074	0.5067	0.5074	0.00186	0.0015	0.001552	0.00156	
		3.5	0.3536	0.3599	0.3646	0.3614	0.3609	0.00157	0.00129	0.001451	0.00134	
		5	0.3015	0.3063	0.3133	0.3094	0.3084	0.00147	0.00122	0.001447	0.00128	
		ALL	-	-	-	-	-	0.00133	0.00112	0.001207	0.00116	

Table (15). Real and estimated reliability values and their associated MSE values when θ = 0.8, β = 0.5 and a sample size of n = 96

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
12	8	0.05	0.9266	0.9296	0.925	0.9273	0.9277	0.00028	0.00019	0.000143	0.00015	WLS
		0.5	0.5743	0.5899	0.581	0.5831	0.5837	0.00182	0.00129	0.00131	0.00137	
		1.5	0.3299	0.342	0.3449	0.3415	0.3411	0.00126	0.00124	0.001162	0.00117	
		3.5	0.1895	0.1955	0.2063	0.1999	0.1988	0.001	0.00116	0.000937	0.00089	
		5	0.1469	0.1511	0.1633	0.1564	0.1552	0.00093	0.00109	0.000834	0.00077	
		ALL	-	-	-	-	-	0.00106	0.00099	0.000877	0.00087	
16	6	0.05	0.9266	0.9258	0.9238	0.9249	0.9255	0.00028	0.0002	0.000232	0.00021	MPS
		0.5	0.5743	0.5741	0.5717	0.5737	0.5744	0.00217	0.00128	0.001335	0.00138	
		1.5	0.3299	0.3256	0.331	0.3289	0.3289	0.00116	0.00072	0.000724	0.00072	
		3.5	0.1895	0.1819	0.1921	0.1876	0.1871	0.00075	0.00045	0.000614	0.00048	
		5	0.1469	0.139	0.1497	0.1449	0.1443	0.00069	0.00039	0.000607	0.00044	
		ALL	-	-	-	-	-	0.00101	0.00061	0.000702	0.00065	

Table (16). Real and estimated reliability values and their associated MSE values when θ = 0.5, β = 1.5 and a sample size of n = 96

m	k	t	Values					MSE				Best
			Real	MLE	MPS	OLS	WLS	MLE	MPS	OLS	WLS	
12	8	0.05	0.9837	0.9825	0.9822	0.9827	0.983	1.72E-05	1.76E-05	1.94E-05	1.61E-05	MPS
		0.5	0.866	0.8597	0.8583	0.8609	0.8627	0.00075	0.00074	0.000832	0.00073	
		1.5	0.7071	0.7003	0.6993	0.7024	0.7042	0.00206	0.00192	0.002197	0.00203	
		3.5	0.5477	0.5435	0.5443	0.5455	0.5461	0.00282	0.00247	0.002771	0.00267	
		5	0.4804	0.4775	0.4794	0.4792	0.4791	0.0029	0.00248	0.002724	0.00267	
		ALL	-	-	-	-	-	0.00171	0.00152	0.001709	0.00162	
16	6	0.05	0.9837	0.9838	0.9833	0.9834	0.9838	1.76E-05	1.54E-05	1.70E-05	1.44E-05	MPS
		0.5	0.866	0.8675	0.8647	0.8652	0.8674	0.0008	0.00069	0.000739	0.00067	
		1.5	0.7071	0.7107	0.7077	0.7078	0.7105	0.00219	0.00185	0.001951	0.00189	
		3.5	0.5477	0.5512	0.5512	0.5499	0.5518	0.00275	0.00238	0.002468	0.00248	
		5	0.4804	0.4829	0.4849	0.4829	0.4842	0.00265	0.00236	0.002435	0.00247	
		ALL	-	-	-	-	-	0.00168	0.00146	0.001522	0.00151	

7- The Applied Side:

Bladder cancer is one of the common diseases that pose a threat to human life. It is also one of the diseases whose degree of reliability cannot be predicted or measured except after some time from the spread of the disease. In this paper, data were obtained for 96 bladder cancer patients, and the data represents. The period of remission of the disease in months (beginning of complete recovery). Data were taken from (Rady, *et al*, 2016).

7-1 Goodness-of-fit tests

In order to ensure that the data represented by the patient’s length of stay in the hospital follow a Lomax distribution, the Kolmogorov-Smirnov (K-S) test, the Anderson Darling (A-D) test, and the Chi-Squared test were used, as the null hypothesis states that the data have a Lomax distribution, as follows:

$$H_0: x \sim Lomax\ Distribution$$

$$H_1: x \nsim Lomax\ Distribution$$

Distribution suitability graphs for the data under study were obtained using the statistical program EasyFit 5.6, and were as follows:

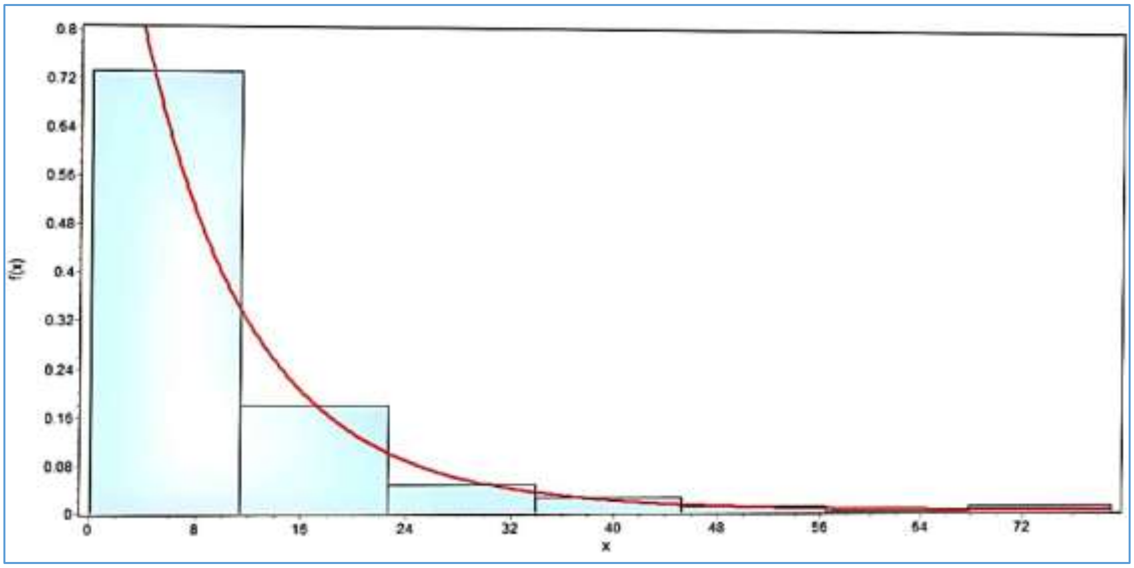

Figure (1) Probability density function curve for the Lomax distribution for real data

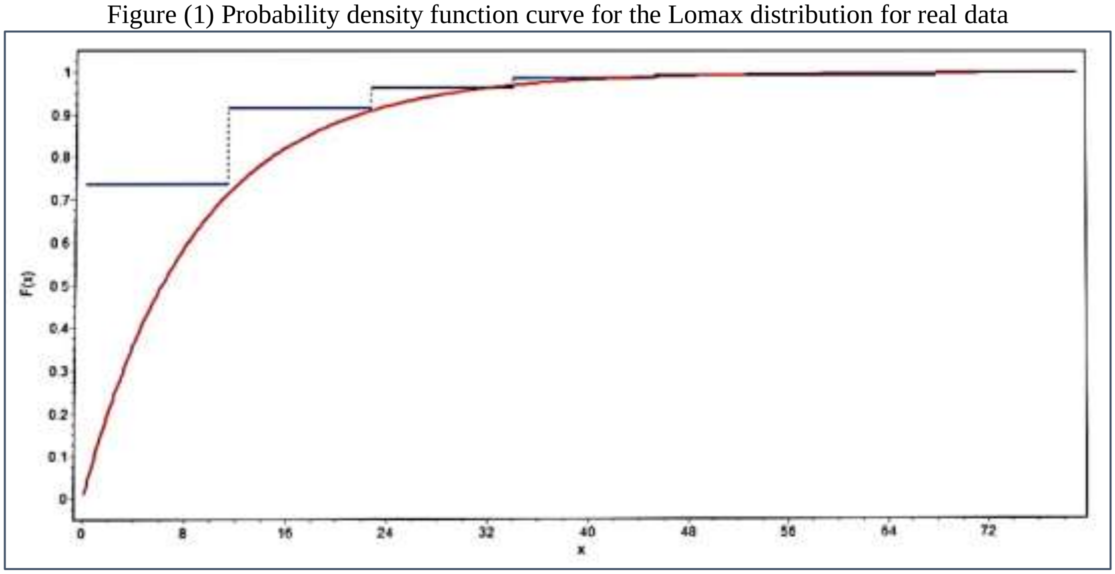

Figure (2) Cumulative distribution function curve for the Lomax distribution for real data

The results of goodness-of-fit tests also indicate that the data has a Lomax distribution, as follows:

Table (17). Chi-square test results for goodness of fit

Test	Calculated value	Critical value	Result
Kolmogorov-Smirnov	0.0967	0.12	follows Lomax distribution
Anderson Darling	1.3768	2.5018	follows Lomax distribution
Chi-Squared	6.1	12.592	follows Lomax distribution

7-2 Estimation using the (RSS) method

RSS technology was used to draw a sample size of 96 from the total sample according to the number of cycles (k=16) and the number of units in each group (m=6), and the drawn sample was as follows:

Table (18). Duration of hospital stay in months for the sample using (RSS)

3.64	9.02	7.66	5.85	7.87	26.31
2.46	3.52	3.31	9.74	10.75	11.79
1.26	3.7	3.36	7.28	5.06	18.1
2.83	7.26	2.64	13.11	5.41	4.33
0.81	3.36	9.74	7.63	13.29	79.05
1.35	2.09	6.97	6.76	3.64	17.14
7.39	0.81	7.28	23.63	12.03	21.73
3.64	1.35	7.59	6.25	14.77	5.34
0.81	1.46	5.49	11.64	7.62	18.1
0.9	1.05	9.02	7.59	17.36	20.28
5.17	9.22	0.81	10.75	10.66	16.62
2.87	0.9	5.09	8.65	7.09	79.05
0.9	4.23	12.07	7.66	9.74	19.13
3.82	5.32	4.26	14.77	16.62	8.37
1.4	6.54	4.34	13.8	14.24	21.73
0.81	5.32	5.41	5.85	17.36	34.26

The two parameters of the Lomax distribution were estimated using the (MPS) method because it is the best method in simulation experiments. The results were ($\hat{\theta}$ =6.563425; $\hat{\beta}$ =55.962806), and based on these estimates the reliability function for this distribution was estimated, as follows:

$$\hat{R}(t) = \left(1 + \frac{t}{\hat{\beta}}\right)^{-\hat{\theta}} = \left(1 + \frac{t}{55.962806}\right)^{-6.563425}$$

based on this equation, the reliability function was estimated at the survival times of the studied data, as follows:

Table (19). Values of the reliability function estimated using the (RSS) method

t	$\hat{R}(t)$	t	$\hat{R}(t)$	t	$\hat{R}(t)$
0.81	0.9100	5.17	0.5599	10.66	0.3184
0.90	0.9006	5.32	0.5510	10.75	0.3156
1.05	0.8851	5.34	0.5498	11.64	0.2893
1.26	0.8640	5.41	0.5457	11.79	0.2851
1.35	0.8552	5.49	0.5411	12.03	0.2786
1.40	0.8503	5.85	0.5207	12.07	0.2775
1.46	0.8445	6.25	0.4991	13.11	0.2512
2.09	0.7861	6.54	0.4841	13.29	0.2470
2.46	0.7540	6.76	0.4731	13.80	0.2354
2.64	0.7389	6.97	0.4628	14.24	0.2258
2.83	0.7234	7.09	0.4571	14.77	0.2150
2.87	0.7202	7.26	0.4491	16.62	0.1815
3.31	0.6858	7.28	0.4481	17.14	0.1731
3.36	0.6820	7.39	0.4430	17.36	0.1698
3.52	0.6701	7.59	0.4340	18.10	0.1589
3.64	0.6613	7.62	0.4326	19.13	0.1452
3.70	0.6569	7.63	0.4322	20.28	0.1314
3.82	0.6483	7.66	0.4309	21.73	0.1161
4.23	0.6199	7.87	0.4216	23.63	0.0991
4.26	0.6178	8.37	0.4006	26.31	0.0797
4.33	0.6132	8.65	0.3893	34.26	0.0435
4.34	0.6125	9.02	0.3750	79.05	0.0031
5.06	0.5666	9.22	0.3675		

5.09	0.5648	9.74	0.3488		

The estimated reliability function has been plotted with the true reliability function as follows:

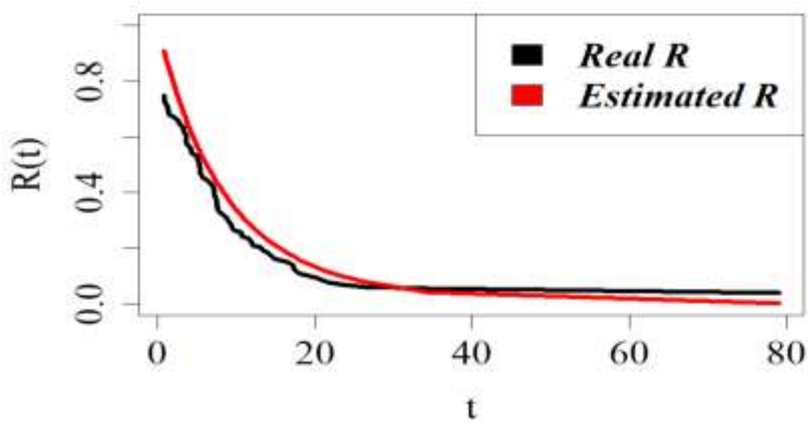

Figure (3). Drawing the true and estimated reliability functions based on the (RSS) method

8. Discussion

The observed and recorded data in the current work were consistent with Lozano *et al*. (17), who found that the onset of puberty and prevalence of estrus behavior started at 5.4 to 6.9 months. In addition to that, the bodyweight data agreed with Ehtesham and Vakili, Nieto *et al.* (18,19) that recorded an increase in body weight at the time of puberty 210-240 days, reaching 35 Kg. Body mass plays a crucial role via changes in growth hormone during aging, especially at the early stages of life till puberty, with the consequence to fat deposition in the body. Fat deposition plays a vital role in puberty and the onset of estrus via increased leptin formation and its release in the body (20). This is logically accepted and supports our finding, which records an increase in leptin level as age progresses and increases body weight (21,22). Therefore, body weight strongly correlates with reproductive health system function and activity in terms of interference with sexual hormones.

9. Conclusion

On the experimental side, and through the results obtained in tables (3 to 16) and through (MSE), it was shown that the best estimation method is based on the ranked set sampling (RSS) method for the Lomax distribution, taking the default parameter values for the distribution θ = (0.8, 0.5, 0.5) and β = (0.5, 0.5, 1.5) is the method of maximum product spacings (MPS) at different sample sizes, which are (12, 24, 36, 54, 96) and by drawing several different groups (m) and repeating several different cycles (c). Based on the results of the experimental side, real data was taken regarding the period of disease remission in months (beginning of complete recovery) for bladder cancer patients and the best method was applied to these data, which confirmed the superiority of the method over the rest of the methods by observing the curves of the reliability function and the aggregate function. From table (19) and figure (3), it is clear that as the healing period was shorter, the reliability was greater with the treatment and recovery protocol

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Conflict of interest

The author has no conflict of interest.

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تقدير موثوقية توزيع لوماكس تحت أخذ العينات مجموعة المرتبة (RSS) مع التطبيق

موسى محمد موسى[1] ، بان غانم العاني[2]

[2،1]قسم الإحصاء والمعلوماتية ، كلية علوم الحاسوب والرياضيات جامعة الموصل ، الموصل ، العراق

الخلاصة: في بعض الأحيان يواجه الباحث مشكلة في الحصول على البيانات ، أو قد يكون هناك صعوبة في الحصول عليها بسبب التكلفة أو الجهد أو لأسباب أخرى تتعلق بالوقت. في هذه الحالة ، يتم استخدام أساليب أخذ العينات التي تضمن الباحث يحقق هدفه المنشود مع وقت قصير والجهد والتكلفة ، وذلك باستخدام أخذ العينات مجموعة المرتبة (رسس). في هذه الورقة ، تم تقدير دالة الموثوقية لتوزيع لوماكس تحت (رسس) باستخدام أربع طرق تقدير ، وهي مقدرات الاحتمالية القصوى (ملي) ، المنتج الأقصى للمسافات (مبس) ، طريقة المربعات الصغرى (لس) ، والمربعات الصغرى المرجحة (ولس). كما تم استخدام طريقة محاكاة مونت كارلو لتحديد أفضل طريقة ، وتم اختيار أفضل تقدير باستخدام معيار الخطأ المربع المتوسط ، وتم تطبيق النتائج في الجانب النظري باستخدام برنامج آر ، حيث أظهرت نتائج المحاكاة أن الطريقة الأكثر فعالية من بين الطرق المستخدمة لتقدير دالة الموثوقية لتوزيع لوماكس تحت (آر إس إس) هي طريقة (مبس). تم تطبيق الجانب التجريبي على البيانات الحقيقية التي تمثل أوقات بداية الشفاء التام (أوقات مغفرة المرض) في أشهر لمرضى سرطان المثانة لعينة تتكون من 96 مريضا تم رسمها باستخدام طريقة (آر إس إس).

الكلمات المفتاحية: مجموعة أخذ العينات المرتبة, توزيع لوماكس, أقصى احتمال, المنتج الأقصى لمقدر التباعد, المربعات الصغرى, المربعات الصغرى الموزونة.